NUMERICAL MODELING OF ELECTRIC ARCS

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Thermal plasmas generated by electric arc discharges between various types of electrodes or by plasmatrons have many well-established — and numerous potential — applications in extractive metallurgy, materials processing and high temperature chemistry. The current is from below 100 A in welding arcs to above 100 kA in electric arc furnaces (EAF) for steel-making and submerged arc furnaces (SAF) for production of silicon alloys. AC as well as DC is used. To improve process understanding and equipment design a number of simulation models have been developed, reaching higher levels of sophistication as more computer capacity has become available. This report reviews the state-of-the-art of arc simulation and discusses some important problems and challenges for future modelling work — in particular on high currents and AC operation. The perspective is the metallurgical and chemical engineers' demand for practical simulation models — not the physicist's very stringent approach.

Introduction. Arc discharges have found widespread applications in various fields of high temperature processing. There is an ever increasing demand for more energy efficient processes and reactors, better process control and lower maintenance costs of electrodes and furnace linings. For this reason, numerical simulation of fluid flow and heat and mass transfer in arcs will probably be more and more important. The aim of this report is to review briefly the development of arc modelling, and to discuss some of the problems that need to be studied more carefully in future modelling work on thermal plasmas. High currents and AC arcs are emphasized.

2. Channel Arc Models. The channel arc models (CAM) do not require the solution of a set of coupled non-linear partial differential equations, and can be run on any PC. In the primitive CAM description the arc column is assumed to be a uniform cylinder except for a short cathode contraction. Here the Lorentz force \( j \times B \) generates a plasma jet, which converts energy from the arc towards the anode.

For a given current \( I \) and arc length \( L \) the uniform channel temperature \( T \) and radius \( R \) are the two main quantities to be determined. The equations to be satisfied are the integral energy balance of the arc and a second equation provided by Steenbeck's energy minimum principle [1]:

\[
P_{\text{el}} = \Sigma \text{heat losses}, \quad \frac{\partial P_{\text{el}}}{\partial R} dR + \frac{\partial P_{\text{el}}}{\partial T} dT = 0
\]

Channel models might be useful for estimating current-voltage characteristics of DC as well as AC arcs considered as non-linear elements in the electric circuits of EAFs and SAFs.

3. Models Based on Differential Conservation Equations. The channel arc models are based on integral balances for mass, momentum and energy. They can give no information about the "inner structure" of the arc. To obtain more detailed information, e.g., the temperature and velocity distributions, the arc and its nearest surroundings must be divided into a large number of small computational cells. The mass, \( x-, y- \) and \( z- \)momentum, energy and electric charge balances for each of these cells must be satisfied. In other words: a system of conservation (or transport) equations in the form of a set of non-linear and nasty coupled partial differential equations must be solved with appropriate boundary conditions. The 3D time-dependent conservation equations can be written in Cartesian tensor form.

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The continuity equation

\[ \frac{\partial \rho}{\partial t} + \sum_{i} \frac{\partial (\rho v_i)}{\partial x_i} = 0 \]  

(2)

\( i \) is a subscript that denotes the three space coordinates and \( v_i \) denotes the three velocity components.

The momentum equations

\[ \frac{\partial (\rho v_i)}{\partial t} + \sum_{j} \frac{\partial (\rho v_i v_j)}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \sum_{j} \frac{\partial \tau_{ij}}{\partial x_j} + F_i \]  

(3)

\( F_i \) is the \( i \)-components of the body forces, e.g., the Lorentz force. The stress tensor is given by

\[ \tau_{ij} = \mu_{\text{eff}} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \mu_{\text{eff}} \delta_{ij} \nabla \cdot \mathbf{v} \]  

(4)

\( \mu_{\text{eff}} \) is the effective viscosity: \( \mu_{\text{eff}} = \mu + \mu_t \), where \( \mu_t \) is the turbulent viscosity. To compute \( \mu_t \) a turbulence model must be used, e.g. the \( k-\epsilon \)-model — see Sect. 8.

The energy equation

\[ \frac{\partial (\rho h)}{\partial t} + \sum_{i} \frac{\partial (\rho v_i h)}{\partial x_i} = \sum_{j} \frac{\partial}{\partial x_j} \left( k_{\text{eff}} \frac{\partial h}{\partial x_j} \right) + S_h \]  

(5)

\( k_{\text{eff}} \) represents the effective thermal conductivity:

\[ \frac{k_{\text{eff}}}{c_p} = \frac{k}{c_p} + \frac{\mu_t}{Pr_t} \]  

(6)

where \( Pr_t \) is the turbulent Prandtl number and \( k \) the molecular thermal conductivity. \( S_h \) is a general source term, which includes ohmic heating, radiation, etc. — see Sect. 7.3.

In addition, conservation (diffusion) equations for chemical species might be included.

3.1. The Prescribed Current Distribution Approximation. To reduce the numerical complexity, various simplifying assumptions were made in earlier works on arc modelling. For free-burning axisymmetric DC arcs in argon a prescribed current density distribution \( j_z(r, z) \) was first used by Ramakrishnan et al. [2]. Analytic expressions are given for the parabolic distribution \( j_z(r) \) and the expanding arc radius \( R(z) \):

\[ j_z(r, z) = \frac{2I}{\pi R^2} \left( 1 - \frac{r^2}{R^2} \right), \quad R(z) = R_{ca} \left( 1 - \frac{z}{R_{ca}^{1/2}} \right) \]  

(7)

where \( R_{ca} \) is the assumed cathode spot radius (e.g. determined by the Richardson–Dushman formula) and \( C \) is an empiric or adjustable "arc expansion factor." The azimuthal magnetic field \( B_{\theta} \) and the radial and axial components of the Lorentz force \( j \times B \) are then obtained analytically from (7):

\[ B_{\theta}(r, z) = \frac{\mu_0 I}{2\pi R^2} r \left( 2 - \frac{r^2}{R^2} \right), \quad F_r = j_z B_{\theta}, \quad F_z = j_r B_{\theta} \]  

(8)

\[ j_r(r, z) = \frac{I}{\pi R^3} C \frac{R_{ca}^{1/2}}{z^{1/2}} r \left( 1 - \frac{r^2}{R^2} \right) \]  

(9)