NUMERICAL ANALYSIS OF BIFURCATION PROBLEM WITH CORANK-$n^*$

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Abstract

In this paper, numerical approximation of solution branches of bifurcation problem with corank-$n$ is studied.

Key words. Bifurcation point, nonlinear equation, the extended system

1. Introduction

Let $E$ be a Hilbert space and $G : E \times R \rightarrow E$ be a $c^3$-continuous nonlinear mapping. Consider the parameter dependent equation

$$G(u, \lambda) = 0. \quad (1.1)$$

Assume that there is a point $(u_0, \lambda_0) \in E \times R$ satisfying

(H1) $G_0 := G(u_0, \lambda_0) = 0.$

Here and in the sequel the subindex $0$ indicates the evaluation of the corresponding function and the operator at the point $(u_0, \lambda_0)$.

(H2) $D_uG_0$ is a Fredholm operator with index 0, and zero is one of its eigenvalues with algebraic multiplicity $N$. Furthermore,

(H3) $\dim (\ker (D_uG_0)) = N$, $D_\lambda G_0 \in \text{Range} (D_uG_0)$.

Under the assumptions (H1)-(H3), equation (1.1) is called an $N$ corank bifurcation problem, and $(u_0, \lambda_0)$ an $N$ corank bifurcation point of it.

In the following, we introduce the solution structure and the extended system of solution branches of an $N$ corank bifurcation point $(u_0, \lambda_0)^[1]$; the symbols $D, D_u, D_\lambda, \cdots$ represent the total and partial derivatives w.r.t. $u, \lambda, (u_1, u_2, \cdots)$ respectively.

Fredholm operator theory shows that there are elements $\varphi_i, \Psi_i \in E, i = 1, 2, \cdots, N$ such that

$$E_1 = \ker (D_uG_0) = \text{span} \{\varphi_1, \varphi_2, \cdots, \varphi_N\}, \langle \varphi_i, \varphi_j \rangle = \delta_{ij},$$

$$E_2 = \text{Range} (D_uG_0) = \{u \mid u \in E \langle u, \varphi_i \rangle = 0, 1 \leq i \leq N\},$$

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\[ E = E_1 \oplus E_2 = \mathbb{E}_1 \oplus \mathbb{E}_2, \]

where \( D_uG_0^* \) is the adjoint operator of \( D_uG_0 \), \( \langle \cdot, \cdot \rangle \) is the dual form or inner product on \( E \times E^* \) and \( \oplus \) represents the direct sum of subspaces in \( E \).

According to assumptions (H3), there exists a unique \( v_0 \in E_2 \) such that

\[ D_uG_0v_0 + D_\lambda G_0 = 0. \]

In the sequel we use the notation

\[
\begin{align*}
q_{ij} &= D_{uv}G_0\varphi_i\varphi_j, \\
q_{00} &= D_uD_G_0(v_0, 1)\varphi_i, \\
q_{00} &= D_\lambda G_0(v_0, 1)^2,
\end{align*}
\]

\[ a^k_j = \langle \Psi_k, q_{ij} \rangle, \]

\[ a^k_0 = \langle \Psi_k, q_{00} \rangle, \]

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Lemma 1.1. Under the assumptions (H1)-(H3), the solution curves of (1.1) can be expressed in the form

\[
\begin{align*}
u(t) &= u_0 + t\alpha^i\varphi_i + t\beta v_0 + t^2v, & v &\in E_2, \\
\lambda &= \lambda_0 + t\beta,
\end{align*}
\]

Lemma 1.2. Assume \( \alpha(t) \in C(I, R^N), \beta(t) \in C(I, R) \). Then \( F(v, \alpha, \beta, t) \) defined by (1.5), (1.6) is continuous at \( t = 0 \).

Theorem 1.1. Under the assumptions (H1)-(H3) to find all solutions curves \( u(t), \lambda(t) \) of (1.1) passing through \( (u_0, \lambda_0) \) is equivalent to finding the nontrivial solution \( (v, \alpha, \beta) \in E_2 \times R^N \times R \) of the equations \( F(v, \alpha, \beta, t) = 0 \).

Usually we cannot obtain exact values of the bifurcation point \( (u_0, \lambda_0) \) and the relevant parameters \( \varphi_i, \Psi_i, v_0 \) \((i = 1, 2, \cdots, N)\), there are only approximations of them available. Therefore, it is necessary to consider the influence of the approximate values, we will discuss the approximation form of the mappings (1.5) and (1.6). In the following, we introduce several elementary results on the finite dimensional approximations of regular solutions of nonlinear problems.\(^2\)

Assume \( X, Y, Z \) are Banach spaces, \( B_x, B_y \) are bounded, open convex subsets of \( X, Y \) respectively and \( \Phi(x, y) : B_x \times B_y \to Z \) is a \( c^r \) mapping \((r \geq 1)\).