EFFECT OF CORRELATIONS OF THE EXPERIMENTAL DATA ON THE ERRORS IN EVALUATED NEUTRON CROSS SECTIONS

S. A. Badikov, E. V. Gai, and N. S. Rabotnov

This work concerns the underestimation of the errors in the recommended neutron cross sections obtained by using strict statistical methods to analyze measurements. It is shown that one reason for this is inadequate representation of the initial experimental data, specifically, correlations of the experimental data are neglected. The effect of correlations of the experimental data on the error in the evaluated neutron cross sections is studied using realistic models and practical examples. Special attention is focussed on correlations of measurements performed by different authors. Analysis of correlations of this kind substantially increases the errors in the evaluated cross sections. Analysis of the model examples shows that the correction can be large (~70%) even for cases with relatively weak correlation of the measurements performed by different authors. For the evaluated cross sections of the reactions $^{103}$Rh(n, n)$^{103m}$Rh and $^{237}$Np(n, f) the corrections reach 41% and 25% for certain energy ranges. 5 figures, 1 table, 22 references.

In the last few years a substantial fraction of the evaluations of neutron reaction cross sections was based on an analysis of experimental data by strict statistical methods: the least-squares method, the Bayesian formalism, and the maximum-likelihood method. However, wide use of these methods for the evaluation of neutron data is held back by the following circumstance. We are talking about the large underestimate of the errors in the recommended neutron cross sections obtained after the measurement results are analyzed by strict statistical methods. An indicative example is the evaluation of the standard – the neutron fission cross section of $^{235}$U, where the errors in the estimated cross section which are extracted by the least-squares analysis of the experimental data were increased by a factor of 2–3 by experts from the US Cross Section Evaluation Working Group [1]. The resulting errors in the evaluated cross sections for some other reactions, for example, the errors in the evaluated cross section for the reaction $^{93}$Nb(n, 2n)$^{92m}$Nb [2], were subjected to well-argued criticism. There are several reasons why the errors of the evaluated cross sections are underestimated. One is an inadequate representation of the initial experimental information, specifically, the correlations in the measurement results are neglected. The physical basis for the correlations in the experimental data is that the same neutron sources, samples, monitoring reactions, and decay constants are used in different measurements.

Our objective in the present work is to study the effect of the correlations of the experimental data on the errors in the evaluated neutron cross sections. Attention will be focused on the correlations of the measurement results obtained by different authors (in modern investigations [3, 4] such correlations are considered to be negligibly small).

Computational Scheme. The calculations were performed by the least-squares method. Rational functions, which are a natural tool for describing neutron cross sections, were used as approximating functions [5]. The computational scheme, presented in detail in [6], is valid for uncorrelated measurements performed by different authors. When they are correlated, only the calculation of the correlation matrix $P$ of the uncertainties in the measurements is subject to change. The elements of the matrix were calculated as
Fig. 1. Effect of the correlation measurements on the error of the evaluated dependence of the cross section of the reaction \((n, p)\) for 14.9 MeV neutrons: 1) measurement errors in the cross section; 2, 3) errors in the estimated dependences of the cross sections calculated taking account of and neglecting the correlations of the measurements, respectively; 4) error in the cross sections of the monitoring reaction.

\[
P_{ij} = \frac{\sum_{k=1}^{K} p_{ij}^k e_i^k e_j^k}{e_i e_j},
\]

where \(e_i^k\) is the \(k\)th systematic component of the error \(e_i\) in the cross section; \(p_{ij}^k\) is the correlation coefficient between the \(k\)th systematic components \(e_i^k\) and \(e_j^k\). The matrix \(P\) can be represented in the form of blocks of matrices of lower dimension \(P = (P_{lm})\), \(l, m = 1, ..., M\), where \(M\) is the number of experiments. The block \(P_{lm}\) was approximated by a matrix with a constant element

\[
P_{lm} = \frac{\sum_{i=1}^{n_l} \sum_{j=1}^{n_m} p_{ij}}{n_l n_m};
\]

where \(n_l\) is the number of measurements in the \(l\)th experiment. For \(l = m\) averaging is performed over the off-diagonal elements.

We note that the least-squares method gives asymptotically unbiased estimates. For this reason the difference of the estimates will be determined by the individual features of the statistical samples whether or not the correlations in the measurements are taken into account. Moreover, the analysis of the correlations in the experimental data is equivalent to a decrease in the initial information. In accordance with the general principle of information theory (the less initial information available, the greater the uncertainty in the resulting information), the uncertainty of the estimates should increase when studying the correlations of measurements in a statistical analysis of experimental data. Therefore, the main object of investigation is the scale of this effect.

**Analysis of the Correlations of Measurements Performed by One Author.** In this and the next section, before real examples are considered, exactly solvable, very simple models will be analyzed. The results and estimates obtained by examining exactly solvable models are of interest as "standards" for understanding the general laws and for analyzing experimental data in real situations. As a first model example we shall consider \(n\) identically correlated measurements \(\sigma_i\) of an unknown average \(\theta\). The measurement error \(\epsilon\) and the correlation coefficient \(\rho\) are known. The calculations of the variance \(V(\bar{\theta})\) of the estimate in the least-squares method for \(\theta\) leads to the expression