APPLICATION OF THE METHOD OF RADIAL AVERAGING TO DESCRIBE DIFFUSION IN A GAS CENTRIFUGE WITH A NONUNIFORM CIRCULATION FLUX

O. E. Aleksandrov

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The theory of separation in a Tsipp gas centrifuge is developed using the method of radial averaging. A more general method for deriving the one-dimensional diffusion equation is proposed. An application of the averaging method for an arbitrary distribution of the circulation flux along the axis of the centrifuge is given. A general solution and a solution for a uniform circulation flux are obtained in a simpler form with the requirement that the concentration of one of the isotopes be small. 1 figure, 3 references.

To increase the efficiency of commercial gas centrifuges it is necessary to know the available margin for increasing their productive capacity. At present, efforts are being focused on producing computational-simulation programs, though the efficiency limits have still not been determined. The efficiency is determined by many factors, one of the most important of which is the internal flow of gas in the separating chamber of the rotor (see Fig. 1). For example, the separating power of a centrifuge

\[ \delta U = PV(N_p) + WV(N_w) - FV(N_F) \]

is usually estimated using the Dirac’s estimate [1]

\[ \delta U_{\text{max}} = \frac{\pi}{2} H D \left( \frac{\Delta m (R \Omega)^2}{2kT} \right)^2, \]  

where \( F, P, \) and \( W \) are the feed, product, and waste fluxes, respectively; \( N_p, N_w, \) and \( N_F \) are the concentration in the feed, product, and waste flows, respectively; \( V \) is the value function \( V(N) = (2N - 1)\ln N/(1 - N); \) \( H \) is the rotor length; \( \rho \) is the density of the gas being separated; \( D \) is the diffusion coefficient; \( \Delta m \) is the difference of the molecular masses of the isotopes \( \Delta m = |m_1 - m_2|; \) \( R \) and \( \Omega \) are the inner radius and angular rotational velocity of the rotor; \( k \) is the Boltzman’s constant; and, \( T \) is the gas temperature.

According to Eq. (1), the separating power of modern centrifuges is several times lower than the maximum possible value. Attempts to reach efficiency comparable to the estimate (1) have been unsuccessful. In addition, the experimental dependence of \( \delta U \) on the rotor length is nonlinear. However, modern centrifuges show that the efficiency is still 10–20% higher than predicted by Cohen type analytical models [2]. This shows that the latter models are inadequate.

The estimate (1) has one other substantial drawback – it does not relate the separating power of a centrifuge with the external (feed, product, and waste) and internal (circulation) fluxes. This means that this estimate is applicable to a certain limiting operating regime, specifically, for an infinitely high feed flux and circulation flux in the rotor. It is obvious that real centrifuges do not work in such a regime, at least because the separation factor approaches zero in this limit. The working fluxes
are chosen on the basis of a compromise between the requirements of stable circulation in the rotor, a finite and, hopefully, large separation factor, and a finite feed flux. Computational models cannot answer such questions, since they do not give a visible picture of the phenomenon because of the large number of parameters and the complexity of the programs themselves. Analytical methods for describing diffusion are not currently being developed.

In the present paper we endeavor to return to the analytical description of separation at a new level – the method of radial averaging – and, first and foremost, to justify its application for an arbitrary distribution of the circulation flux in the rotor.

The method of [2] reduces the problem of searching for a two-dimensional distribution of the concentration in the centrifuge rotor to a one-dimensional problem of the concentration distribution along the axis. The advantage of the method is that it permits deriving simple analytic relations between the gas flow in the rotor and the efficiency, i.e., the general laws of the effect of the gas flow can be investigated. The original derivation [2] of the diffusion equation substantially simplified the picture of gas flow in the rotor:

it was assumed that the circulation flux in the rotor does not vary along the centrifuge and that the flux closes in negligibly thin layers near the ends of the rotor; and

the feed flux was assumed to be negligibly small compared with the circulation flux in the rotor.

This led to the incorrect belief that the one-dimensional diffusion equation [2] is inapplicable for quantitative description of separation in the presence of a large nonuniformity of the circulation flux along the rotor and a finite feed flux. Moreover, it was asserted a priori that any radial flow degrades the efficiency of the centrifuge because of convective mixing [1].

We shall show that the one-dimensional diffusion equation can be obtained without assuming that the radial mass flow in the centrifuge is zero, and in this case the accuracy of the description of diffusion is not degraded.

We shall use the coordinate system and notation shown in Fig. 1. Cylindrical coordinates are used. It is assumed that \( r = 0 \) corresponds to the rotation axis of the centrifuge and \( z = 0 \) is the feed point. The problem is assumed to be two-dimensional. The isotopic approximation is considered, i.e., it is assumed that the concentration of the isotope does not influence the motion of the gas as a whole.

We divide the flow inside the centrifuge rotor into two components: 1) the component associated with circulation (rotational flow, closed streamlines) and 2) the component associated with the feed flow (potential flow, open streamlines). Mathematically, this can be done by decomposing the velocity vector field into rotational and irrotational parts. Such a decomposition exists and is unique for any continuous vector field [3]:

\[
pV = pV_\psi + pV_\varphi = pD\text{rot}(\Psi) + \text{grad}(\varphi),
\]

where \( \Psi = \begin{pmatrix} 0 \\ \Psi \\ 0 \end{pmatrix} \) is the vector potential of the velocity and characterizes the intensity of circulation; \( \varphi \) is the scalar potential characterizing the direct flow; \( V_\psi \) and \( V_\varphi \) are the flow velocities; the factor \( pD \) is introduced for convenience, and it is assumed to be coordinate-independent. The only nonzero component of \( \Psi \) is due to the two-dimensionality of the flow.

In the special case of a no separation regime \( (F = 0) \) and no radial mass flows \( (pV_r = 0) \), the stream function \( \psi \) [2] (see [1]) and the circulation potential \( \Psi \) are related as \( pD\Psi r = \psi \). In other cases, the circulation potential \( \Psi \) is different from the stream function \( \psi \). The main difference is that the feed flux does not make a direct contribution to \( \psi \). This does not mean that the feed flux has no effect on the circulation in the rotor. The feed flux is once again considered as a source of energy (together with the temperature gradient and mechanical excitation by the separator) for excitation of rotational motion of gas in the rotor. But, in addition, it produces a special type of flow – a direct flow which is irrotational and must be studied separately.

Neglecting the existence of two types of flow in a rotor [2] is justified by the assumption that the feed flux is small compared with the circulation flux. In modern centrifuges the feed flux is comparable to the circulation flux.

The isotope mass flux can also be represented similarly to the mass flux:

\[
\Phi = \text{rot}(\Psi D) + \text{grad}(\varphi D).
\]

The vector potential \( \Psi D \) determines the rotational isotope mass flux and the scalar potential \( \varphi D \) determines the direct isotope mass flux in the rotor. By definition the isotope mass flux is