CREEPING WAVES ON A HIGHLY ELONGATED BODY OF REVOLUTION

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Creeping waves play an important role in diffraction by a smooth convex body and give an asymptotics of the diffracted field in the shadow. Known results obtained by the boundary-layer method do not allow us to explain some of the properties of creeping waves on highly elongated bodies. In this paper, creeping waves on highly elongated bodies are studied in the case where the binormal curvature of the surface is asymptotically large. The asymptotics derived contains solutions of a differential equation of the Heun type. The analysis of the dispersion equation for the surface waves is carried out numerically. It is discovered that the magnetic creeping wave travels along the surface of a highly elongated body with much less attenuation than predicated by the usual theory. Bibliography: 5 titles.

1. INTRODUCTION

Creeping waves providing an asymptotics of the diffracted field in deep shadow play an important role in problems of diffraction. Known results obtained by the boundary-layer method do not explain some properties of the creeping waves on highly elongated bodies. Mathematically, the problem lies in the construction of a high-frequency asymptotics of the solution of Helmholtz or Maxwell equations that satisfies given boundary conditions. Earlier, when examining problems of diffraction, the sizes of a body were assumed to be comparable in different directions, and asymptotic expansions were constructed in powers of a single small parameter \( k^{-1/3} \), where \( k \) is the wave number. If the sizes of a body differ significantly in different directions, an additional large parameter, namely, the ratio of these sizes, occurs in the statement of the problem.

The goal of the present paper is to study creeping waves on highly elongated bodies when the binormal curvature of the surface is asymptotically large. The solutions of [1] show that in the case of an ordinary, unelongated body, the binormal curvature manifests itself only in the correction term of the asymptotics of acoustic waves on a rigid body and of a magnetic transversal wave in the case of Maxwell equations. If the curvature in the binormal direction is large and is comparable with \( k^{1/3} \), then the correction given in [1] is of the same order as the principal term and the asymptotic expansion is not valid. Thus, in the case of a highly elongated body, an additional analysis of diffraction problems is required.

2. GEODESIC COORDINATES AND THE ANSATZ

Let a surface of revolution be defined in cylindrical coordinates \( (r, \varphi, z) \) by the equation \( r = r(z) \). In the case of a highly elongated body, the function \( r(z) \) varies slowly. We assume that

\[
r, r'(z) \ll 1.
\]

In the half-plane \( \varphi = 0 \), introduce an arc-length \( s \) and a normal \( n \) to the surface. The asymptotic behavior of the creeping waves is obtained in the coordinate system \( (s, \varphi, n) \). In order to rewrite the Helmholtz equation and a system of Maxwell equations in the coordinates \( s, \varphi, n \), we need the matrix \( g_{ij} \) of quadratic forms. Usually (see, e.g., [3]), to construct the leading term of the asymptotics, it is sufficient to know only the linear terms in the expansion of this matrix in powers of \( n \). In our case, we need the exact expression

\[
g_{ij} = \begin{pmatrix}
(1 + \frac{n}{\rho})^2 & 0 & 0 \\
0 & \rho^2 (1 + \frac{n}{\rho})^2 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

Here, and $\rho$ and $\rho_t$ are the principal radii of surface curvature; the quantity $h$ characterizes the divergence of the pencil of geodesics and is proportional to the radius $r$ of the body. The radii of curvature are expressed in terms of the function $r(z)$ by the formulas

$$\frac{1}{\rho(s)} = \frac{\tau_{zz}}{(1 + r_z^2)^{3/2}}, \quad \rho_t = \frac{1 - r_z^2}{1 + r_z^2}.$$  

We assume that the scale of the $z$ axis and, respectively, of the $s$ axis is chosen so that $\rho(s) = O(1)$. Restrictions (1) imply the smallness of $\rho_t$. We assume that the normalized radius of binormal curvature

$$\kappa = 2M^2\rho_t/\rho, \quad M = \left(\frac{k\rho}{2}\right)^{1/3},$$

is of order $O(1)$.

The asymptotic behavior of the field of creeping waves (acoustic pressure in the scalar diffraction problem and all the components of electric and magnetic vectors in the problem of diffraction for Maxwell equations) is sought in the standard [3] form of the boundary-layer method:

$$u = \exp\{iks + i\kappa^{1/3}p(s, \varphi)\} \sum U_j(s, \varphi, \nu_1)\kappa^{-j/3}, \quad \nu_1 = k^{2/3}n.$$  

(3)

Here, $\nu_1$ is the stretched normal defining the thickness of the boundary layer. The functions $U_j$ are assumed to have no rapid dependence on their arguments, i.e., all the derivatives of $U_j$ are considered of order $O(1)$.

The time factor $\exp(-i\omega t)$ is omitted throughout the paper.

As shown in [2], significant physical effects manifest themselves for a highly elongated body when the ratio of the radii of curvature $\rho/\rho_t$ is comparable with the square of the large parameter $M = \left(\frac{k\rho}{2}\right)^{1/3}$. In this case, the whole cross-section of the body lies in the boundary-layer domain and the formulas lose their local character in the variable $\varphi$. In our case of a body of revolution, we expand the field into the Fourier series in the angular variable and assume below that the field is dependent on $\varphi$ via the factor $e^{im\varphi}$.

### 3. SCALAR WAVES

Substituting expression (3) into the Helmholtz equation rewritten in the coordinates $(s, \varphi, n)$ and equating the terms of the same order in $k$ yield a system of recurrent equations successively determining the functions $U_j$. The ratio $n/\rho_t$ in the boundary layer is of order one. As a result, the binormal curvature occurs in the first equation of the recurrent system

$$L_0 U_0 = 0, \quad L_0 U_1 + L_1 U_0 = 0 \quad \ldots,$$

where the operators take the form

$$L_0 = \frac{\partial^2}{\partial \nu^2} + \frac{1}{\nu} \frac{\partial}{\partial \nu} + \left(\nu - \frac{m^2}{\nu^2} - \xi\right),$$

(4)

$$L_1 = \left(\frac{\rho}{2}\right)^{2/3} \left(2i \frac{\partial}{\partial s} + i \frac{\partial \rho_t/\partial s}{\rho_t/\nu} \kappa\right).$$

(5)

Here, the shifted normal coordinates are introduced as follows:

$$\nu = 2M^2\rho_t + \frac{n}{\rho},$$

and the parameter $\xi$ is connected with the function $p(s)$ in (3) by the formula

$$p(s) = 2^{-1/3} \int_0^s \frac{\xi - \kappa}{\rho^{2/3}} ds.$$