THE EDGE WAVE IN THE PROBLEM OF DIFFRACTION ON A BOUNDARY WITH JUMP OF CURVATURE

V. B. Philippov and N. Ya. Kirpichnikova

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The problem of the diffraction of creeping waves on a point of transition of the convex boundary to the straight boundary of a domain is investigated. It is assumed that at the point of jump of curvature, the tangent to the boundary is continuous and its derivative has a jump. An expression for the edge wave is obtained and investigated. Bibliography: 4 titles.

In the present paper, the problem of the diffraction of a creeping wave on the boundary of a domain is considered. We assume that the boundary consists of two parts: convex and flat. Denote the convex part of the boundary by $S^-$, the flat part by $S^+$, and the curve that is an analytic extension of the curve $S^-$ by $S_{im}^+$ (see Fig. 1).

Introduce the coordinates $(x, y)$ with origin at the point $O$ of jump (discontinuity) of curvature; the $x$ axis is directed along the boundary $S^+$ and the $y$ axis is directed along the outer normal to the domain; we also introduce semigeodesic coordinates $(s, n)$ near $S^-$ and $S_{im}^+$ so that $n = 0$ on the boundary $S^-$ and $S_{im}^+$, $s < 0$ on $S^-$, and $s > 0$ on $S_{im}^+$. At the origin $O$, $s = n = 0$ for $x = y = 0$.

We assume that a creeping wave $u^-(x, k)$, where $x = (n, s) \in \Omega^-$, propagates along the boundary $S^-$. In a neighborhood of the point $O$, we introduce the stretched coordinates

$$
\sigma = \frac{M}{\rho}, \quad \nu = 2M^2 \frac{n}{\rho} \tag{1}
$$

and

$$
X = \frac{M}{\rho}, \quad Y = 2M^2 \frac{y}{\rho} \tag{2}
$$

where $M = (\frac{k_0}{\rho})^{1/3}$, and $\rho$ is the radius of curvature of the curve $S^-$ at the point $s = 0$.

For the Dirichlet problem in a neighborhood of the point $O$, the creeping wave can be written as follows:

$$
u^-(s, n) = A_0 e^{iks} e^{i\zeta_1 \sigma} w_1(\zeta_1 - \nu), \tag{3}
$$

where $\zeta_1$ is the first root of the Airy function $w_1$.

In [1], a formula for the extension of field (3) to the domain $\Omega^+$ (for $s > 0$) was obtained. It has the form

$$
u^+(x, y) = A_0 e^{iks} e^{i\pi/4} \left( e^{i(y-y')^2/4} - e^{i(y+y')^2/4} \right) w_1(\zeta_1 - Y') dY'. \tag{4}
$$

It was shown that the functions $u^+$ and $u^-$ coincide on the line $x = s = 0$ and the derivative has a jump on this line, i.e., we have

1. $u^+ (+0,0) = u^- (-0,0)$,

2. $\left.\left(\frac{\partial u^+}{\partial x}\right)\right|_{+0} - \left(\frac{\partial u^-}{\partial x}\right)\left|_{-0}\right. \neq 0$.

From physical considerations and also from the analysis of similar model problems (see [2, 3], where diffraction on a jump of curvature in the illuminated and penumbra regions is investigated), we may expect that in a more complicated case of diffraction on a jump of curvature in the shadow region, there exists an edge wave that spreads from the line of a jump of curvature. The present paper is concerned with the examination of this edge wave.

Formulas (3), (4) give a representation for the field in a neighborhood of the boundary in the first approximation. These formulas give rise to formulas for the current on the boundary in the first approximation, namely,

$$\Psi^-(s) = \left(\frac{\partial u^-}{\partial n}\right)_{n=0} = B_0 e^{iks} e^{i\zeta_1 \sigma},$$

$$\Psi^+(s) = \left(\frac{\partial u^+}{\partial y}\right)_{y=0} = B_0 \frac{2 e^{i(kX+\pi/4)}}{w_1'(\zeta_1)} \int_0^\infty e^{i\frac{y^2}{4X}} w_1'(\zeta_1 - Y) dY,$$

where

$$B_0 = A_0 w_1'(\zeta_1) \frac{\partial \nu}{\partial n} = A_0 w_1'(\zeta) \frac{2M^2}{\rho}.$$

The proof of formula (6).

Differentiating formula (4) with respect to $y$, for $y = 0$ we have

$$\left(\frac{\partial u^+}{\partial y}\right)_{y=0} = A_0 \frac{i^{3/2} e^{iks}}{\sqrt{\pi X}} \int_0^\infty Y' e^{i\frac{y^2}{2X}} w_1'(\zeta_1 - Y') dY'.$$

Taking into account that

$$\frac{\partial}{\partial Y'} \left( e^{i\frac{y^2}{2X}} \right) = \frac{iY^2}{2X} e^{i\frac{y^2}{2X}}$$

and using the formula of integration by parts, we obtain formula (6). The term outside the integral is equal to zero, because $\zeta_1$ is the first root of the Airy function $w_1$.

Denote the current generated by formulas (3), (4) as follows:

$$\Psi_0(s) = \left\{ \begin{array}{ll}
\Psi^-(s) = B_0 e^{iks} \Psi^-(\sigma), & s < 0, \\
\Psi^+(x) = B_0 e^{iks} \Psi^+(X), & x > 0 (s > 0),
\end{array} \right.$$ (7)

where

$$\Psi^-(\sigma) = e^{i\zeta_1 \sigma},$$

$$\Psi^+(X) = \frac{2 e^{i\pi/4}}{w_1'(\zeta_1)} \int_0^\infty e^{i\frac{y^2}{2X}} w_1'(\zeta_1 - Y) dY.$$ (9)

We show that

1. $\Psi^+(+0) = \Psi^-(0)$,

2. $\left.\left(\frac{\partial \Psi^+}{\partial X}\right)\right|_{+0} - \left(\frac{\partial \Psi^-}{\partial X}\right)\left|_{-0}\right. \neq 0$.

The proof of formulas (10), (11).