TRANSFORMATION OF LASER RADIATION BY CENTERED SYSTEMS OF LENSES

A. P. Khapalyuk

The general properties of transforming the parameters of simple Gaussian beams by systems of centered lenses are investigated in the general case with arbitrary complex focal lengths. Analytical and graphic methods for solving the problems of this transformation are proposed.

Key words: complex-focal-length lenses, complex transformer, Gaussian beam, linear-fractional transformation.

In laser physics, conventional optical lenses and Gaussian diaphragms are widely used to transform laser-beam parameters. However, the capabilities of the individual real lenses and Gaussian diaphragms are rather limited. Systems that consist of a certain number of these elements and whose optical properties were not investigated systematically before offer greater capabilities. Therefore, the need arises for investigating the general mechanisms of transformation of the parameters of the simplest Gaussian beams (GBs) by a finite number of centered lenses in the general case with complex focal length.

In what follows, it is assumed that the two-dimensional simple GBs which occur here in the Cartesian coordinate system x, z are written in ordinary form [1]:

$$E = A(t, z) \exp \left( \frac{i k^2 x^2}{2} \frac{1}{a - i k w^2 - z} \right),$$

where \(a\) determines the position of the locus (the narrowest portion) for a GB; \(w\) is its radius at the locus; \(k\) is the wave number; \(A\) is the amplitude factor which will not be of interest in what follows. We consider \(a\) and \(k w^2\) to be the diagnostic GB variables, and the study of the characteristics of a variation in them as GB passes through the lens systems is the aim of this work.

Let there be \(N\) centered lenses on the z axis at points \(z_j (j = 1, 2, ..., N)\) with complex focal lengths. The process of GB passing (1) through this system can be considered as a complex process that consists of simple processes of GB passing successively through each lens individually. The beam transformed by the first lens can be considered to be incident to the second lens, etc. The lens properties are modelled by the transfer function [2]

$$M_j(x) = \exp \left( i \frac{k^2 x^2}{2} \frac{1}{F_j} \right).$$

On each lens, ordinary boundary conditions (equality of the tangential components of the field vectors) must be satisfied, which lead to the following chain of equations [1]:

$$\frac{1}{a_j - i k w_j^2 - z_j} = \frac{1}{a_{j-1} - i k w_{j-1}^2} + \frac{1}{F_j}.$$

---

We obtain \( N \) coupled complex nonlinear equations relative to \( 2N + 2 \) real parameters: \( a_0, a_1, \ldots, a_N \) and \( kw_0^2, kw_1^2, \ldots, kw_N^2 \). We need to assign two of them; the remaining \( 2N \) are found from Eq. (3) which determines the concrete and mathematically rigorous statement of the problem. Of greatest practical interest are two variants of this problem:

1) the direct problem of analysis when the parameters of the incident GB \((a_0, kw_0^2)\) are considered to be assigned parameters, while the parameters of the transmitted GB \((a_N, kw_N^2)\) are considered to be sought parameters; 2) the inverse problem of analysis — the parameters of the transmitted GB are considered to be assigned parameters while the parameters of the incident GB are considered to be sought parameters. The parameters of intermediate beams (that propagate between the lenses) are of less interest, but we can find them when required. As the first step in the solution of the problem, we can consider eliminating from system (2) the intermediate-beam parameters and obtaining the formula that links the parameters of the GB incident to the system of lenses and the GB emerging from it. In this case, it is appropriate to introduce one complex parameter \( G_i = a_i - ikw_i^2 \) for GB instead of two real parameters. In this notation, with a little manipulation, Eq. (3) can be written in the form

\[
G_j = \frac{(z_j + F_j) G_{j-1} - z_j^2}{G_{j-1} - (z_j - F_j)}.
\]

Expression (4), for all \( j \), has the same form and mathematically is the formula of linear-fractional transformation [3, 4]. This is a conformal one-to-one mapping of the points of an extended (including the infinite point) complex plane of the variable \( G_{j-1} \) onto the points of the same plane of the variable \( G_j \). In mathematics, the general regularities of this transformation are known and will be allowed for in what follows. In essence, they should be specified for the real physical processes considered here, and interpreted in conventional optical terminology with allowance made for the established concepts of laser physics.

Equalities (4) can be simplified further if notation

\[
C_{j1} = z_j - F_j, \quad C_{j2} = z_j + F_j,
\]

is introduced which makes it possible to arbitrarily write them in matrix form:

\[
G_j = \frac{C_{j2} G_{j-1} - (z_j^2 + C_{j1} C_{j2})}{G_{j-1} - C_{j1}} = \begin{pmatrix} C_{j2} & - (z_j^2 + C_{j1} C_{j2}) \\ 1 & - C_{j1} \end{pmatrix} G_{j-1} = M_j G_{j-1}.
\]

The rules of constructing matrix \( M_j \) from the coefficients of linear-fractional transformation (4) are evident, and GB parameter \( G_{j-1} \) is multiplied by matrix \( M_j \) according to the rules that follow from identities (6). In what follows, we will define the physical sense of parameters (5). The presentation introduced here is convenient primarily leading to a simple rule for elimination of unnecessary parameters of intermediate GBs.

To eliminate from the first two equations of (6) the parameter of the intermediate GB \( G_j \), we need to substitute its value from the first \((j = 1)\) equality into the second equality \((j = 2)\) which leads to the following expression:

\[
G_2 = \begin{pmatrix} C_{22} - F_2^2 - C_{22} C_{21} \\ 1 - C_{21} \end{pmatrix} \begin{pmatrix} C_{12} - F_1^2 - C_{12} C_{11} \\ 1 - C_{11} \end{pmatrix} G_0 = M_2 M_1 G_0 =
\]

\[
= \begin{pmatrix} C_{22} (C_{12} - C_{21}) - F_2^2 - C_{11} C_{22} (C_{12} - C_{21}) - C_{22} F_2^2 + C_{11} F_2^2 \\ C_{12} - C_{21} \end{pmatrix} G_0.
\]

The product of matrices \( M_2 M_1 \) is performed according to conventional rules of matrix algebra which justifies the notation introduced.

Successively continuing similar operations, we can easily obtain the required formula that completes the first step of the problem: