ON AN IDENTITY FOR DUAL FIELDS

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A new identity for dual fields is proved. Bibliography: 4 titles.

INTRODUCTION

In the present paper, we consider some questions on the algebraic Bethe ansatz connected with the method of dual fields. Auxiliary quantum operators (dual fields) were first introduced in [1]. Via this construction it became possible to obtain compact determinant formulas for correlation functions of quantum integrable models (see, e.g., [2]).

It has already been mentioned in the first paper devoted to the method in question that, in some cases, the dual fields do not contribute to the final result. This observation was based on comparison of the formulas obtained in the framework of the dual-field method with the results obtained by other approaches. However, a direct proof of this fact was not given. In the present paper, we establish an identity that allows us to exclude the auxiliary quantum operators from determinant formulas that describe some important special cases of scalar products.

1. DUAL FIELDS

We consider a generalized integrable model which can be solved by means of the algebraic Bethe ansatz. The monodromy matrix of such a model

\[ T(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix} \] (1.1)

possesses a highest vector \(|0\rangle\) having the following properties

\[ A(\lambda)|0\rangle = a(\lambda)|0\rangle, \quad D(\lambda)|0\rangle = d(\lambda)|0\rangle, \quad C(\lambda)|0\rangle = 0. \] (1.2)

The dual vector \(<0|\) has similar properties,

\[ <0|A(\lambda) = a(\lambda)<0|, \quad <0|D(\lambda) = d(\lambda)<0|, \quad <0|B(\lambda) = 0. \] (1.3)

The explicit form of the functions \(a(\lambda)\) and \(d(\lambda)\) involved in (1.2) and (1.3) is not not essential.

As usual, the commutation relations for the entries of the monodromy matrix are given by the \(R\)-matrix:

\[ R(\lambda, \mu) \left( T(\lambda) \otimes T(\mu) \right) = \left( T(\mu) \otimes T(\lambda) \right) R(\lambda, \mu), \] (1.4)

where

\[ R(\lambda, \mu) = \begin{pmatrix} f(\mu, \lambda) & 0 & 0 & 0 \\ 0 & g(\mu, \lambda) & 1 & 0 \\ 0 & 1 & g(\mu, \lambda) & 0 \\ 0 & 0 & 0 & f(\mu, \lambda) \end{pmatrix}. \] (1.5)

We shall consider the case of the rational \(R\)-matrix:

\[ f(\mu, \lambda) = \frac{\mu - \lambda + ic}{\mu - \lambda}, \quad g(\mu, \lambda) = \frac{ic}{\mu - \lambda}. \] (1.6)
Here \( c \) is an arbitrary constant, whose physical sense depends on the concrete model. It is also convenient to introduce the following two functions:

\[
\begin{align*}
    h(\mu, \lambda) &= \frac{f(\mu, \lambda)}{g(\mu, \lambda)} = \frac{\mu - \lambda + ic}{ic}, \\
    t(\mu, \lambda) &= \frac{g(\mu, \lambda)}{h(\mu, \lambda)} = \frac{c^2}{(\mu - \lambda)(\mu - \lambda + ic)}.
\end{align*}
\]

The eigenfunctions of the transfer matrix \( \tau(\lambda) = \text{tr} T(\lambda) = A(\lambda) + D(\lambda) \) (which are also common eigenfunctions of the integrals of motion of the model under consideration) can be constructed by means of the operators \( B(\lambda) \):

\[
|\Psi_N(\{\lambda\})\rangle = \prod_{j=1}^{N} B(\lambda_j)|0\rangle, \quad N = 0, 1, \ldots
\]

Similarly, one can construct the dual eigenfunctions via the operators \( C(\lambda) \):

\[
(\Psi_N(\{\lambda\})| = (0) \prod_{j=1}^{N} C(\lambda_j), \quad N = 0, 1, \ldots
\]

In both cases, the parameters \( \{\lambda\} \) must satisfy the system of Bethe equations

\[
a(\lambda_j) \sum_{k=1}^{N} \frac{f(\lambda_j, \lambda_k)}{f(\lambda_k, \lambda_j)} = 1, \quad j = 1, \ldots, N. \tag{1.8}
\]

The investigation of correlation functions in the framework of the algebraic Bethe ansatz is based on the following properties of the scalar products:

\[
S_N(\{\mu\}, \{\lambda\}) = \langle 0 | \prod_{j=1}^{N} C(\mu_j) \prod_{j=1}^{N} B(\lambda_j)|0\rangle. \tag{1.9}
\]

Here \( \{\mu\} \) and \( \{\lambda\} \) are two sets, each consisting of \( N \) arbitrary complex numbers \( (N = 1, 2, \ldots) \). A rather nice and compact expression for the scalar products in terms of the mean value of the determinant of an \( N \times N \) matrix depending on auxiliary quantum operators (dual fields) was found in [1]. We present a brief description of this construction.

Consider four operators \( p_A(\lambda), p_D(\lambda), q_A(\lambda), \) and \( q_D(\lambda) \) depending on the complex variable \( \lambda \), acting in the auxiliary Fock space, and satisfying the conditions

\[
p_{A,D}(\lambda)|0\rangle = 0, \quad (0|q_{A,D}(\lambda) = 0.
\]

Nonzero commutation relations are given by the formulas

\[
[p_A(\lambda), q_A(\mu)] = \ln h(\mu, \lambda), \\
[p_D(\lambda), q_D(\mu)] = \ln h(\lambda, \mu).
\]

The vacuum vector \( |0\rangle \) is normalized to unity. The dual fields \( \Phi_A(\lambda) \) and \( \Phi_D(\lambda) \) are linear combinations of the operators \( p \) and \( q \):

\[
\Phi_A(\lambda) = q_A(\lambda) + p_D(\lambda), \\
\Phi_D(\lambda) = q_D(\lambda) + p_A(\lambda).
\]

It is easy to check that the dual fields commute with each other.