OPTIMAL ESTIMATION OF THE APPEARANCE TIME OF A PULSED SIGNAL WITH RANDOM SUBSTRUCTURE

A. P. Trifonov and O. V. Chernoyarov

An algorithm for optimum estimation of the appearance time of a pulsed signal with random substructure, which is observed against the background of white Gaussian noise, is synthesized. The comparative analysis of the Bayesian and maximum likelihood meters of the appearance time is performed. The results of statistical modeling of estimation algorithms are presented.

1. INTRODUCTION

In many applications of statistical radiophysics it is required to determine the appearance time of the pulsed signal. A Bayesian algorithm for estimating the time of appearance of a quasideterministic pulse of known shape, which is observed against the background of white Gaussian noise, is discussed in [1]. Using the methods of statistical modeling, its working capacity and sufficiently high efficiency were determined. However, under the actual conditions of generation, propagation, and reception of radiowaves, the radiopulse structure can be distorted significantly due to the noise and random action of propagation medium [2--4]. Therefore, it is of importance to solve the problem of synthesis and analysis of algorithms for estimating the appearance time of a pulsed signal with random substructure. The maximum likelihood estimation (MLE) of the appearance time of a pulsed signal with random substructure is considered in [3].

Asymptotic [with an unlimited increase in the signal-to-noise ratio (SNR)] properties of the MLE with allowance for anomalous errors are considered in [3]. However, it is known that the problems of signal processing are solved more efficiently by the Bayesian approach [1, 5, 6] and can ensure an efficiency that is higher than the maximum likelihood (ML) algorithms. Therefore, it is of interest to synthesize and analyze the Bayesian meter of the appearance time of the random pulsed signal. Below, within the framework of the Bayesian approach [1, 5, 6], we obtain a sufficiently simple and efficient algorithm for estimating the appearance time of a radio pulse with random substructure.

2. ESTIMATION OF THE APPEARANCE TIME OF A PULSED SIGNAL WITH RANDOM SUBSTRUCTURE

Let in the time interval \([0; T]\) an additive mixture between a pulsed signal with random substructure \(s(t, \lambda_0)\) and noise \(n(t)\) be observed

\[
z(t) = s(t, \lambda_0) + n(t).
\]

Here the appearance time \(\lambda_0 \in [\Lambda_1, \Lambda_2]\) is assumed to be a random quantity with an a priori probability density \(W(\lambda)\).

In accordance with [2--4], a pulsed signal with random substructure is assumed to be a section of realization of the random process with sufficiently long duration \(\tau\)

\[
s(t, \lambda_0) = \xi(t) I[(t - \lambda_0)/\tau].
\]

In Eq. (2) \(I(\cdot)\) is the indicator of unit duration and \(\xi(t)\) is realization of the stationary centered Gaussian random process. The spectral density of the process \(\xi(t)\) describing the random substructure of the pulsed signal is written as [7]...
Here $\gamma$ and $\Omega$ are the value and the width of the frequency band, respectively, and $\vartheta$ is the center frequency of the spectral density. The function $g(x)$ describes the form of the spectral density and is normalized such that $\max g(x) = 1$ and $\int g^2(x)\,dx = 1$. It is assumed that the random pulsed signal (2) is completely within the observation interval, i.e. $0 < \Lambda_1 - \tau/2 < \Lambda_2 + \tau/2 \leq T$. By analogy with [1, 3], the noise $n(t)$ in Eq. (1) is approximated by a white Gaussian noise with one-sided spectral density $N_0$. On the basis of the observed realization (1), we should estimate the random time of appearance of a radio pulse with random substructure (2).

It is assumed that the duration $\tau$ of the pulsed signal (2) is much greater than the correlation time of the process $\xi(t)$, i.e., $\mu = \tau \Omega/2\pi \gg 1$. Then, in accordance with [3, 4], the logarithm of the functional of the likelihood ratio has the form

$$L(\lambda) = \frac{\lambda}{M(\lambda) - N_0 - (\tau \Omega/2\pi) \int_{-\infty}^{\infty} \ln[1 + g g(z)]\,dz},$$

$$M(\lambda) = \int_{\lambda - \tau/2}^{\lambda + \tau/2} y^2(t)\,dt,$$

where

$$\gamma = \gamma/N_0,$$

and $y(t)$ is the response of the filter with transfer function $H(\omega)$ to the realization of the observed data (1). In this case, the condition $|H(\omega)|^2 = f[|\vartheta - \omega|/\Omega] + f[|\vartheta + \omega|/\Omega]$, $f(x) = g g(x)/(1 + g g(x))$ is fulfilled.

In accordance with [5], the MLE $\hat{\lambda}$ of the appearance time of the random pulsed signal (2) is determined as the location of the absolute (greatest) maximum of the functional $M(\lambda)$ (4)

$$\hat{\lambda} = \arg \sup_{\lambda \in [\Lambda_1, \Lambda_2]} M(\lambda),$$

From Eq. (6) it is obvious that the MLE structure is independent of the a priori probability density of parameter $\lambda_0$.

The block-diagram of the ML meter of the appearance time of a pulsed signal with random substructure is shown in Fig. 1 by a dashed line, where 1 is the switch that is open for time $[\Lambda_1 - \tau/2, \Lambda_2 + \tau/2]$; 2 is a filter with transfer function $H(\omega)$; 3 is the squarer; 4 is the integrator; 5 is the delay line for time $\tau$; 6 is the extremator that fixes the location of the greatest maximum of the signal as an estimate of $\hat{\lambda}$.

The estimate accuracy is characterized by the unconditional variance (standard error) of the appearance time

$$V(\hat{\lambda}) = \int_{\Lambda_1}^{\Lambda_2} V(\hat{\lambda}|\lambda) W(\lambda)\,d\lambda,$$

where $V(\hat{\lambda}|\lambda_0)$ is the conditional variance of the MLE of the appearance time of the pulsed signal (2).

The expression $m = (\Lambda_2 - \Lambda_1)/\tau$ is the reduced length of the a priori interval of possible values of the appearance time of the random signal [6]. Obviously, $m$ is the number of nonoverlapping signals (2), which can be located in the interval $[\Lambda_1, \Lambda_2]$. If $m \gg 1$, then, in accordance with [5, 6], the conditional variance of the MLE $\hat{\lambda}$ (6) is determined by the approximate formula

$$V(\hat{\lambda}|\lambda_0) = P_0 V_0(\hat{\lambda}|\lambda_0) + (1 - P_0) \left[ \frac{\Lambda_2^3 + \Lambda_2 \Lambda_1 + \Lambda_1^3}{3} - (\Lambda_2 + \Lambda_1)\lambda_0 + \lambda_0^2 \right].$$

717