DYNAMICS OF EXCITATION PULSES IN TWO COUPLED NERVE FIBERS*

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In this paper, we study the phenomena of interaction of traveling pulses in a system of two coupled nerve fibers. Each fiber is modeled by a discrete chain of mutually coupled Fitz Hugh-Nagumo excitabile elements. The interaction among the fibers can be distributed, homogeneous, and also localized at certain points in space. In the case of homogeneous interaction, we show the possibility of interfiber synchronization of pulses. For localized interactions, we study the dynamics of point and two-point contacts, allowing us to perform effective control over the propagation of excitation along the coupled fibers.

1. INTRODUCTION

A nerve fiber is very similar to an electric cable and consists of a conducting core, i.e., intracellular protoplasm, surrounded by a shell, i.e., fiber membrane [1-4]. Under certain conditions, an excitation pulse can propagate along the nerve fiber. The formation and propagation of a nerve pulse are based on variation of the local conductivity of the fiber membrane. The membrane is an active device creating nonequilibrium distributions of sodium and potassium ions on opposite sides of the membrane due to chemical processes.

One of the features of the nervous system of highly developed animals is the formation of bundles of coupled nerve fibers [3, 5], which allows much more information to be transmitted per time unit by nerve pulses compared with that transmitted by a single fiber. For example, the sciatic nerve of a rabbit consists of approximately 400 so-called myelinated fibers. Each fiber of the above type is a thin fiber covered by an isolating shell (myelin), and the membrane functions normally only at small sections (the so-called annular isthmuses). The fibers in a bundle are coupled due to a synaptic contact (electric contact, in the simplest case) between the branches (terminals) of the adjacent fibers. It is obvious that the most important problem concerning bundles of nerve fibers is the study of interfiber interactions.

Note that the physiological phenomenon of interaction of pulses in adjacent nerve fibers was discovered in 1940 [3]. Different aspects of the study of the dynamics of interfiber interactions of excitation pulses are discussed in [6-10].

This paper mainly deals with the influence of the structure of interfiber coupling on the dynamics of excitation pulses. The fact that fibers have annular isthmuses, i.e., spatially separated nodes, speaks of the substantial inhomogeneity of the medium. One of the simplest methods for modeling such a medium is the use of a discrete chain of interacting excitabile elements. In addition, using such a model, we can easily introduce the interfiber interaction, which is exactly of a discrete nature from the physiological viewpoint.

We discuss a system of two coupled discrete chains of excitabile Fitz Hugh-Nagumo elements and describe the different phenomena of penetration of excitation pulses between the latter. Moreover, we demonstrate that the discrete nature of both the fibers and interaction between them can lead to new nontrivial phenomena of excitation circulation between the fibers.

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2. THE MODEL

Let us consider two chains of electrically coupled FitzHugh-Nagumo excitable elements and introduce the interaction between the latter via an “electrical” contact of each pair of corresponding elements, as shown in Fig. 1. The dynamics of such a system is described by the following equations:

\[
\begin{align*}
\dot{x}_j^{(1)} &= f(x_j^{(1)}) - y_j^{(1)} + D_1 \Delta x_j^{(1)} + h_j(x_j^{(2)} - x_j^{(1)}), \\
\dot{y}_j^{(1)} &= b(x_j^{(1)} - \gamma y_j^{(1)}), \\
\dot{x}_j^{(2)} &= f(x_j^{(2)}) - y_j^{(2)} + D_2 \Delta x_j^{(2)} + h_j(x_j^{(1)} - x_j^{(2)}), \\
\dot{y}_j^{(2)} &= b(x_j^{(2)} - \gamma y_j^{(2)}),
\end{align*}
\]

where \((\cdot)^{(1)}\) and \((\cdot)^{(2)}\) correspond to the variables of the first and second fiber, respectively, \(\Delta x_j = x_{j-1} - 2x_j + x_{j+1}\), \(D_1\) and \(D_2\) are the coupling coefficients between the elements inside each fiber, \(h_j\) is the vector of interfiber interaction, \(f(x) = x(x - a)(1 - x)\), and \(0 < a < 1\). For each fiber we use the zero boundary conditions \(x_0^{(i)} = x_{N+1}^{(i)} = 0\), \(i = 1, 2\).

2.1. Pulses in an individual fiber

Let us first consider an individual isolated fiber. Its dynamics is determined by Eq. (1) for \(h_j = 0\). The choice of parameters allows the excitation pulses to propagate in such a system. For this purpose we fix the parameters \(b\) and \(\gamma\) (\(b = 0.004\) and \(\gamma = 3\)) and assume that the coupling coefficients \(D_i\) are sufficiently large (\(D_i > D^{(0)}\) and \(D^{(0)}\) is some critical value). The parameter \(a\) is chosen as a check parameter that changes the excitation threshold of the fiber. In this case, Eq. (1) is similar to the distributed FitzHugh-Nagumo equation (long-wave approximation), which is known to have solutions in the form of traveling pulses of the relaxation type propagating against the “background” of a locally stable unperturbed spatially homogeneous state [1, 3, 5, 11].

2.2. Coupled fibers

Let \(h_j \neq 0\) and an “electric” contact exist between the fibers at all or some discrete nodes. By analogy with [12], we show that in the case in which the interfiber interaction is rather strong, i.e., upon fulfillment of the condition

\[
h_j > h^{(0)} \equiv \frac{1 - a + a^2}{6}, \quad \forall j = 1, \ldots, N
\]