ESTIMATION OF THE GLOBAL STABILITY DOMAIN IN A CHAIN OF CORRELATED PHASE-LOCKED SYSTEMS BY THE METHODS OF IMAGE RECOGNITION THEORY

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We formulate the problem and present the results of estimation of the global stability domain of the equilibrium state in a chain of correlated phase-locked systems by the methods of image recognition theory.

A mathematical model of the chain of correlated phase-locked (PL) systems is the system of differential equations

$$\frac{d\varphi_n}{dt} + \sin \varphi_n = \gamma - \delta \sin \varphi_{n-1} - \chi \sin \varphi_{n+1}, \quad n = 1, N. \quad (1)$$

The equilibrium state of this system corresponds to the onset of a PL mode in all the elements of the chain. We consider the problem of extraction of a global stability domain of the equilibrium state in a limited parameter region $A(0 < \delta < 2.5, 0 < \chi < 2.5, 0 < \gamma < 2)$ by the methods of image recognition theory. Such a problem was considered in [4, 5] to extract in the parameter space both the global stability domain of the equilibrium state and the parameter regions with constant structure of the phase space of nonlinear dynamical systems of the second and third orders. In the present paper, this method is used to estimate the global stability domain of the equilibrium state in a dynamical system with high-dimensional phase space.

In terms of image recognition theory, the problem of extraction of the global stability domain can be interpreted as the problem of dividing the assigned parameter region into two classes: class 1, covering the parameter region in which the equilibrium state is globally stable, and class 2, covering the parameter region with other structures of the phase space. Obtaining estimates requires the formation of a learning set — the set of points belonging to the known class of parameters. The estimate of the boundary of the global stability domain is the interface

$$W(a) = w_1 f_1(\delta) + w_2 f_2(\chi) + w_3 f_3(\gamma) + w_4,$$

where $f_1(\delta)$, $f_2(\chi)$, and $f_3(\gamma)$ are functions of the first or second order. The interface can be found by well-known calculation procedures [2], using the vectors of the learning set and correcting the coefficients $w_i$.

Since the form of the interface is not known, we will choose the points of parameters for the learning set at the nodes of a uniform grid, doubling the number of nodes in each parameter sequentially. For each point of parameters, we study the structure of the phase space using the following principle. Since each coordinate is periodic, we consider the region $D(0 \leq \varphi_n \leq 2\pi), n = 1, 2, \ldots, N$, in the phase space. Owing to the fact that self-intersecting trajectories cannot exist in the phase space, the interfaces between the attraction regions of different stable manifolds necessarily intersect one rib of the polyhedron $D$. Since the
form of the boundaries of the attraction regions is not known, we will choose the initial points with constant spacing on the polyhedron ribs and determine the type of a limiting set of chosen geometry by numerical integration. If the limiting set of all the trajectories is the equilibrium state, then the point of parameters belongs to class 1. If the limiting set of at least one trajectory is not the equilibrium state, then the point of parameters belongs to class 2.

The quality of estimation of the structure of the parameter region $A$ will be characterized by the error probability for at least one stage of learning, classification or choice of points in the parameter region:

$$P_{er} = 1 - (1 - P_{r})^{v}(1 - P_{ge})^{v}(1 - P_{cor}).$$

Here, $v$ is the number of points in the learning set.

It was proved in [5] that as the length of the learning set increases, the quantity $P_{er}$ reaches a minimum