ON A CHARACTERISTIC FEATURE OF ELECTROMAGNETIC WAVE ATTENUATION BY A SPHERICAL PARTICLE IN ABSORBING MEDIUM

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We analyze attenuation of a plane electromagnetic wave by a homogeneous spherical particle in absorbing medium. Relations between the values of imaginary and real parts of the complex refractive index of the medium, which result in negative values of attenuation in the Rayleigh approximation, are obtained for the particular case of a nonabsorbing spherical cavity in a medium with absorption. We give an example of negative attenuation due to small air bubbles in water in the range of centimeter waves.

The scattering of a plane electromagnetic wave by a spherical particle in an absorbing medium has some specific features caused by the fact that in the presence of absorption not only the particle itself but also the surrounding medium take part in the formation of the scattering properties [1, 2]. Unlike in the traditional Mie theory in which the medium that surrounds the particle is assumed to be transparent, allowing for absorption, we must correct some concepts of the Mie theory [2]. One of the most interesting differences between the Mie theory for an absorbing medium and the traditional theory is the possibility of negative values of attenuation caused by a particle. This was observed previously in [1, 2]; however, in the above papers no due attention is paid to analysis of the features of negative attenuation. In this paper we attempt to fill this gap partially. In [2], with attenuation defined as the difference in the power recorded by a receiver in the absence and presence of a particle between the source and detector placed in the far zone of the particle and attenuation cross-section defined as the shading of the aperture area of the receiver due to the presence of the particle, the following expression was obtained for the attenuation cross-section in the absorbing medium:

\[ C_{\text{ext}} = 2\pi \text{Re} \left[ \sum_{l=1}^{\infty} (2l+1) \left( \frac{a_l + b_l}{k^2} \right) \right]. \]  

The coefficients \( a_l \) and \( b_l \) in Eq. (1) have the following form

\[ a_l = \frac{n_1 \Psi_l'(p_1) \Phi_l'(p) - n \Psi_l'(p_1) \Phi_l(p)}{n_1 \Psi_l'(p_1) \xi_l(p) - n \Psi_l'(p_1) \xi_l(p)}, \]

\[ b_l = \frac{n \Psi_l'(p_1) \Phi_l'(p) - n_1 \Psi_l'(p_1) \Phi_l(p)}{n \Psi_l'(p_1) \xi_l(p) - n_1 \Psi_l'(p_1) \xi_l(p)}, \]

where \( p = kR, \ p_1 = k_1R, \ k = 2\pi n/\lambda, \) and \( k_1 = 2\pi n_1/\lambda, \) where \( \lambda \) is the wavelength of incident radiation in vacuum, \( R \) is the radius of the spherical particle, and \( n \) and \( n_1 \) are the complex refractive indices for the material of the medium and the particle. It is evident that expressions for the coefficients \( a_l \) and \( b_l \) in Eq. (1) formally coincide with the expressions for coefficients in the Mie theory, i.e., they are equally expressed via the Riccati-Bessel functions and permittivity of the material of the medium and the sphere. However, unlike in the Mie theory, the refractive index of the surrounding medium in these expressions is a complex value \( n = n' + in'' \). For the nonabsorbing medium, the wave number \( k \) in the medium surrounding the particle becomes real and we can take it out of the sign of the real part in Eq. (1) obtaining thereby the well known expression for the attenuation cross-section from the Mie theory.

Let us consider the behavior of Eq. (1) in the Rayleigh limit. Using the expansion of the Riccati-Bessel functions for $|\rho| \ll 1$ and $|\rho_1| \ll 1$, from Eq. (1) we obtain

$$k_{\text{ext}} = \frac{6\pi}{\lambda} \text{Im} \left[ n \left( \frac{m^2 - 1}{m^2 + 2} \right) \right],$$

where $k_{\text{ext}} = C_{\text{ext}} \left( \frac{4\pi R^2}{3} \right)$ and $m = n_1 / n$. Let us find the conditions for which $k_{\text{ext}} < 0$ for the particular case of a spherical nonabsorbing cavity in an absorbing medium. Since in this case $n'_1 = 1$ and $n''_1 = 0$, we have $m = 1/n$ and, consequently,

$$k_{\text{ext}} = \frac{6\pi}{\lambda} \left[ n' \text{Im} \left( \frac{1 - n^2}{1 + 2n^2} \right) + n'' \text{Re} \left( \frac{1 - n^2}{1 + 2n^2} \right) \right].$$

Rewriting Eq. (5) in the form

$$k_{\text{ext}} = -\frac{12\pi n''}{\lambda |1 + 2n^2|^2} \left[ n'' + \left( \frac{5 + 4n'^2}{2} \right) n'' + \left( \frac{2n''^2 + n'^2 - 1}{2} \right) \right],$$

we see that the sign of $k_{\text{ext}}$ is determined by the sign of the value in brackets. Denoting $z = n'^2$, $b = \frac{5 + 4n''^2}{2}$, and $c = \frac{2n''^2 + n'^2 - 1}{2}$ and taking into account that $k_{\text{ext}} < 0$ for $y = z^2 + bz + c > 0$, we easily find the conditions relating the real and imaginary parts of the complex refractive index of the surrounding medium for which this is valid. The roots of the quadratic trinomial $y$ are $z_1 = -\left( n'^2 + \frac{5}{4} \right) - \frac{1}{4} \sqrt{32n''^2 + 33}$ and