ON THE THEORY OF SHORT LIGHT PULSE PROPAGATION
IN A TURBULENT MEDIUM

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We consider the problem of propagation of light pulses in a turbulent medium. Within the
framework of the used approximation, we obtain a formula for the average intensity of the pulsed
signal. Using this formula, we estimate the time delay of the pulse and its broadening. We find
the characteristic time scales determining the light pulse transformation in a turbulent medium.
The physical meaning of these time scales is explained qualitatively. The quantitative estimates
for the Earth’s atmosphere show that the phenomena in question are significant only for very
short, picosecond pulses.

During the propagation of pulsed signals in a turbulent medium, even in the absence of dispersion, one
can observe the transformation of the wave field which leads, first, to the group (time) delay and broadening
of the pulses (pulse duration increase). The exact solution of this problem has not yet been found and
different approximate methods are used in the literature (see, for example, [1–4]). However, the majority of
them, being rather complicated, do not even clarify what physical parameters are most important in this
problem. This is related to the fact that either numerical methods or asymptotic approximations are usually
used, which give no universal analytical description.

In this paper, keeping in mind that our approach is somewhat restricted, we estimate the group
delay and broadening of pulsed signals on the basis of the average intensity, using the method of smooth
perturbations (MSP) with reference to the propagation of a Gaussian pulse. It is assumed that the MSP
is sufficient for describing the phenomena over almost the entire variation range of parameters since, as is
shown below, the main role in this case is played by the phase fluctuations, which are adequately described
by the MSP [5].

We should be careful with the physical interpretation using Gaussian pulses, which are convenient for
calculations, but have infinite “tails.” Strictly speaking, the actual problem with noise should be considered,
and the time at which the pulse intensity is equal to the noise level intersection should be taken as the
pulse onset (or, accordingly, as its end). However, if the goal is to clarify the basic physical laws and the
characteristic scales of the problem rather than to obtain an exact quantitative solution for the particular
pulse shape, we can confine ourselves to the above idealized formulation, being somewhat careful when
passing to the limiting cases.

1. Let an electromagnetic pulse with a plane phase front propagate along the z axis in a turbulent
medium. We present the field \( U(z, t) \) on the axis in the form of an expansion in terms of monochromatic
harmonics \( \tilde{U}(z, \omega) \) as follows:

\[
U(z, t) = \int \tilde{U}(z, \omega)e^{-i\omega t} d\omega.
\]  

Let us express \( \tilde{U}(z, \omega) \), as is usually performed in the MSP, in terms of the field \( \tilde{U}_0(z, \omega) \) in a uniform
medium and the complex phase \( \Psi \) [5]:

\[
\tilde{U}(z, \omega) = \tilde{U}_0(z, \omega)\exp\{\Psi(z, \omega)\},
\]  

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\[ \Psi(z, \omega) = \chi(z, \omega) + iS(z, \omega), \]  

(2b)

where \( \chi \) and \( S \) are, respectively, the fluctuations of the amplitude (level) logarithm and monochromatic wave phase at the frequency \( \omega \).

Let us estimate the pulse transformation in a statistically uniform turbulent medium at time \( t \) on the basis of the average intensity written as

\[ \langle I(z, t) \rangle = \langle U(z, t)U^*(z, t) \rangle, \]

(3)

where the angle brackets denote averaging over the realization ensemble. Using Eqs. (1) and (2), we find that \( \langle I(z, t) \rangle \) has the form

\[ \langle I(z, t) \rangle = \int d\omega_1 \int d\omega_2 \hat{U}_0(z, \omega_1)\hat{U}^*_0(z, \omega_2)e^{-i(\omega_1-\omega_2)t}\Gamma_2(z, \omega_1, \omega_2), \]

(4)

where we introduced a frequency-separated coherence function of the second order in a turbulent medium, which is written in the form

\[ \Gamma_2(z, \omega_1, \omega_2) = \langle \exp\{\Psi(z, \omega_1) + \Psi^*(z, \omega_2)\}\rangle. \]

Therefore, our task is reduced to finding the function \( \Gamma_2 \).

2. We assume that the complex phase

\[ \Psi(z, \omega_i) \equiv \Psi_i = \chi_i + iS_i \]

is normally distributed. In addition, we use the relations resulting from the second approximation of the MSP [6]:

\[ \langle \chi_i \rangle = -\langle \chi_i^2 \rangle, \quad \langle S_i \rangle = -\langle \chi_i S_i \rangle. \]

(6)

Keeping in mind the well-known result for the Gaussian characteristic function

\[ \langle \exp(\zeta) \rangle = \exp\left\{ \langle \zeta \rangle + \left( \langle \zeta^2 \rangle - \langle \zeta \rangle^2 / 2 \right) \right\} \]

(7)

and using Eqs. (2b) and (6), we obtain

\[ \Gamma_2(z, \omega_1, \omega_2) = \exp\{-D_{\psi^*}(\omega_1, \omega_2)/2 + i\theta(\omega_1, \omega_2)\}. \]

(8)

Here the structural function of the complex phase has the form

\[ D_{\psi^*}(\omega_1, \omega_2) = \langle |\Psi_1 - \Psi_2|^2 \rangle, \]

(9)

and the imaginary term in the exponential function is written as

\[ \theta(\omega_1, \omega_2) = \langle \chi_1 S_2 \rangle - \langle \chi_2 S_1 \rangle. \]

(10)

To find \( D_{\psi^*} \) and \( \theta \), we use the following expression given in [5] for the complex phase of the plane wave with the wave number \( k = \omega/c \), where \( c \) is the velocity of light in free space

\[ \Psi(z, \omega) = \frac{ik}{2} \int_0^z \int d\zeta' d\kappa \exp\left\{ \frac{k^2 (z - \zeta')} {2ik} \right\} g_\varepsilon(\zeta', \kappa). \]

(11)

The stochastic spectrum \( g_\varepsilon \) in the above equation is thus related to the fluctuations of permittivity \( \varepsilon \) in the