We develop a method for reconstruction of two-dimensional distribution of brightness temperature using the measured antenna temperature. The method is based on the Tikhonov theory of ill-posed problems. The efficiency of the method is analyzed by numerical simulations of single- and multiple-beam synthetic-aperture radiometric systems. The proposed method is applied to helicopter measurements of thermal radio emission from the soil.

1. INTRODUCTION

Determination of the actual distribution of radio brightness temperature using the measured antenna temperature is a topical problem of radio astronomy and remote sensing of natural media. The antenna-temperature distribution is a two-dimensional convolution of the actual distribution of radio brightness and angular pattern of the antenna system. Such a convolution results in smoothing (smearing) of the actual distribution of the thermal emission. If the shape of the antenna pattern is known (usually it can be measured with rather high accuracy), one can formulate an inverse problem of reconstruction of radio brightness using the measurement data with accuracy limited by the diffraction resolution.

This problem is reduced to a two-dimensional convolution Fredholm integral equation of the 1st kind. It is known that such an equation is ill-posed, i.e., unstable with respect to infinitesimal measurement errors. To solve such an equation, one should use additional (a priori) information on the desired rigorous solution. It is the specific form of this information that determines the choice of different regularization methods (see [1-3]). For example, iterative algorithms for “image cleaning” [4-8], based on smoothing of high-frequency components of the solution, are widely used in radio astronomy. Some methods, e.g., the least-square method, optimal Wiener filtration [2], or maximum-entropy (statistical-regularization) method [9-12], are based on statistical properties of the rigorous solution. There is also an approach called singular-system analysis, in which expansion of the solution over eigenfunctions is applied [3, 13]. In some cases, a “common sense” regularization is used, in which the solution instability is eliminated by smoothing, by using sufficiently rare numerical mesh, etc. In many papers, a simpler one-dimensional problem was considered for regularization.

In this paper, we apply the Tikhonov principle of generalized residual [1] in which rather general information of square integrability of the desired function and its derivatives is used.

2. INITIAL RELATIONS

If a two-dimensional distribution of brightness temperature on the $xy$ plane, where $x$ and $y$ are Cartesian coordinates, is scanned by an antenna with a known angular pattern, a two-dimensional distribution
of the antenna temperature is obtained. Consider a problem of reconstruction of a radio brightness temperature using the temperature distribution of an antenna with known angular pattern. The antenna and brightness temperatures are related via two-dimensional convolution

\[ K_h T_b = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_h(x - s, y - t) T_b(s, t) \, ds \, dt = T_a^\delta(x, y), \quad (1) \]

where the kernel \( K_h(w, \Omega) \) is the angular pattern of the antenna, \( T_a^\delta(x, y) \) is the measured antenna temperature, and \( T_b(s, t) \) is the desired brightness-temperature distribution. Equation (1) is a two-dimensional convolution Fredholm equation of the 1st kind. Solution of this equation with respect to \( T_b(s, t) \) allows one to reconstruct the brightness-temperature distribution. The error measure \( \delta \) for measured antenna temperature \( T_a^\delta \) and error measure \( h \) for the kernel of Eq. (1) satisfy the following inequalities [1]

\[ \| T_a^\delta - T_a \|_{L^2} \leq \delta, \quad \| K - K_h \|_{W_2^2} \leq h, \quad (2) \]

where the antenna temperature \( T_a \) corresponds to the rigorous solution. According to the generalized-residual principle [1], an approximate solution \( T_b^\alpha \) of Eq. (1) is derived by minimizing the functional of generalized residual

\[ M_a[T_b] = \| K_h T_b - T_a^\delta \| _{L_2^2}^2 + \alpha \| T_b \| _{W_2^2}^2. \quad (3) \]

The minimization is performed under the additional condition that constitutes the generalized residual principle:

\[ \| K_h T_b^\alpha - T_a^\delta \| _{L_2^2}^2 = (\delta + h \| T_b^\alpha \|)^2, \]

i.e., the residual should be equal to the sum of error measures of measurement and kernel. This condition stipulates a one-to-one relation between degree of smoothing of the approximate solution, which is determined by the value of the regularization parameter \( \alpha \) in Eq. (3), and the sum of error measures of measurement and kernel. It is shown in [1] that the parameter \( \alpha \) goes monotonically to zero as the error measure decreases, and thus the influence of smoothing on the approximate solution is gradually diminished. This allows one to reconstruct cumbersome profile of brightness temperature if the accuracy is high enough. The superscript in the notation \( T_b^\alpha \) for the approximate solution points out the correspondence between this solution and the regularization parameter determined according to the generalized residual principle.

Using the properties of the Fourier transforms, one can obtain analytical solution for a convolution equation:

\[ T_b^\alpha(s, t) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{K}_h^\alpha(\omega, \Omega) \tilde{T}_a^\delta(\omega, \Omega) e^{i(\omega s + \Omega t)} \, d\omega \, d\Omega \quad (4) \]

Here \( \tilde{K}_h^\alpha(\omega, \Omega) = \tilde{K}_h(-\omega, -\Omega), \quad L(\omega, \Omega) = |\tilde{K}_h(\omega, \Omega)|^2, \quad \tilde{T}_a^\delta(\omega, \Omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{T}_a^\delta(x, y) e^{-i(\omega x + \Omega y)} \, dx \, dy, \quad (5) \]

and

\[ \tilde{K}_h(\omega, \Omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} K_h(u, w) e^{-i(\omega u + \Omega w)} \, du \, dw. \quad (6) \]

The above-described method has an important advantage. In particular, standard algorithms of the fast two-dimensional Fourier transforms can be applied to numerical realization of this method. This allows one to overcome the well-known difficulties of the solution of a two-dimensional problem on a high-dimension mesh.