CONDITIONS FOR THE EXISTENCE OF WEAKLY DIVERGENT BUNDLES IN PLANE-LAYERED WAVEGUIDES

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We found conditions for the existence of weakly divergent bundles (WDB) of rays of different types in waveguides with a power-law dependence of the square of the refractive index.

It is well known that weakly divergent bundles can appear in refractive plane-layered waveguides near the respective singular rays. Such singular rays correspond to smooth extrema in the angular dependences of the ray cycle length.

Measurements and calculations [1, 2] show that weakly divergent bundles are localized in space and have a quasi-periodic spatial structure. The intensity of the field in the bundle vicinity is considerably greater than the intensity of the ambient field. For a smooth variation of characteristics along the waveguide, the bundles are transformed, changing their structure [3]. The internal structure of the bundle is described by diffraction effects [4–6].

Besides the elevated intensity, the bundles are also characterized by a relatively high coherence both in their excitation by partially coherent sources and in the presence of randomly distributed inhomogeneities inside the waveguide [7]. These properties of the bundles are interesting from the viewpoint of their possible use for the development of tomographic ocean-monitoring schemes [4, 6, 7].

Some important properties of the bundles have not been studied in sufficient detail. In particular, for their practical use it is important to learn to predict the formation of bundles under actual conditions (for example, in oceanic waveguides). In the present paper, we found the conditions of existence of WDBs for a wide range of refractive waveguides with a power-law dependence of the square of the refractive index.

For a piecewise-linear approximation of the refractive index profile across the waveguide (along the coordinate z) the conditions of existence of WDBs were discussed in [3]. The criteria of existence of WDBs for n²-bilinear refractive index profiles were obtained earlier in [7].

1. INTEGRAL EXPRESSIONS FOR THE RAY CYCLE

Consider a stratified waveguide in the region \(-h_2 \leq z \leq h_1\), \(-h_2 < 0 < h_1\), filled with a medium with refractive index \(n = n(z)\). Let \(D(a)\) denote the cycle of the ray escaping at a grazing angle \(\theta_S\) from the source located on the horizon \(z_S\), \(-h_2 < z_S < h_1\), and \(a = n(z_S)\cos\theta_S\) be the normalized phase velocity preserved along the ray.

**Definition.** The ray corresponding to the parameter \(a\) generates a weakly divergent bundle if the function \(D(a)\) has a local extremum at the point \(a\) and the derivative \(D'(a) = 0\).

We notice that this definition is somewhat narrower than that adopted in [3] where the existence of a WDB is determined by the condition \(dD/d\theta_S = D'(a)n(z_S)\sin\theta_S = 0\). The singular ray \(\theta_S = 0\) always satisfies the latter condition but does not necessarily satisfy the condition \(D'(a) = 0\); hence, it may not generate a WDB in the sense of the definition adopted here.

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The function \( D(a) \) has the well-known integral presentation \[8\]

\[
D(a) = 2 \int_{Z_{	ext{min}}}^{Z_{	ext{max}}} \frac{a \, dz}{\sqrt{n^2(z) - a^2}},
\]

where \( z_{\text{min}} = z_{\text{min}}(a) \) and \( z_{\text{max}} = z_{\text{max}}(a) \) are correspondingly the minimal and maximal \( z \)-coordinates of the ray points (rotation horizons of the ray). The functions \( z_{\text{min}}(a), z_{\text{max}}(a) \) are the corresponding solutions of the equation

\[
n(z) = a
\]

if these solutions lie in the interval \([-h_2, h_1]\) or take the values of the boundaries of that interval. Besides (1), we use the following expression for the ray cycle:

\[
D(a) = \frac{d}{da} \phi(a),
\]

where

\[
\phi(a) \equiv 2 \int_{z_{\text{min}}}^{z_{\text{max}}} \sqrt{n^2(z) - a^2} \, dz,
\]

which is easily derived from (1) with allowance for (2) by differentiation of the integral with respect to the parameter \( a \).

<table>
<thead>
<tr>
<th>Ray type</th>
<th>Conditions in ( z_{\text{min}} ) and ( z_{\text{max}} )</th>
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</thead>
<tbody>
<tr>
<td>1st</td>
<td>( z_{\text{min}} &gt; -h_2 ) ( z_{\text{max}} &lt; h_1 )</td>
</tr>
<tr>
<td>2nd</td>
<td>( z_{\text{min}} &gt; -h_2 ) ( z_{\text{max}} = h_1 )</td>
</tr>
<tr>
<td>3rd</td>
<td>( z_{\text{min}} = -h_2 ) ( z_{\text{max}} &lt; h_1 )</td>
</tr>
<tr>
<td>4th</td>
<td>( z_{\text{min}} = -h_2 ) ( z_{\text{max}} = h_1 )</td>
</tr>
</tbody>
</table>

As in [7], we relate the ray in a plane-layered waveguide to one of the four types in accordance with the position of the ray rotation horizons (Table 1). In [3] it is shown that WDBs cannot belong to the fourth type. Type 3 WDBs are not very interesting since they can be broken by waveguide wall inhomogeneities during their reflection. Taking this into account, we concentrate efforts on seeking conditions for the existence of type 1 and type 2 WDBs.

2. WDBS IN ONE-CHANNEL WAVEGUIDES WITH POWER-LAW PROFILES \( n^2 \)

Let the profile \( n^2 \) have a unique maximum in the channel axis of a refractive waveguide. We write \( n^2(z) \) in the form

\[
n^2(z) = n_0^2 - \varphi^2(z),
\]

where \(-h_2 \leq z \leq h_1\), \(-h_2 < 0 < h_1\), and \( n_0 = n(0) \) is the maximum value of the refractive index that can be achieved in the channel axis. Assume that \( \varphi(z) \) is a monotonically increasing and, therefore, reversible function. Let \( \psi(t) \) be a function that is inverse of \( \varphi(z) \). Then, after replacement of variables \( z = \psi(st) \), \( s \equiv s(a) \equiv \sqrt{n_0^2 - a^2} \) we find for the function (4) the following integral presentation:

\[
\phi(a) = s^2 \int_{\varphi(z_{\text{min}})/s}^{\varphi(z_{\text{max}})/s} \psi'(st) \sqrt{1 - t^2} \, dt.
\]

For rays of the first type \( \varphi(z_{\text{min}}) = -s \), \( \varphi(z_{\text{max}}) = s \). In this case, (5) transforms to the relationship