PROBABILITY CHARACTERISTICS OF THE ABSOLUTE MAXIMUM OF GENERALIZED RAYLEIGH STOCHASTIC PROCESS

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We obtain the limiting laws of distribution of the absolute maximum of the generalized Rayleigh random process. Using the methods of statistical modeling, we show that the asymptotic approximations are in good agreement with the actual distributions over a wide range of parameters of the random process.

1. INTRODUCTION

The problem of distribution of the statistical characteristics of non-Gaussian random processes is widely used in statistical radiophysics and radio engineering [1-9]. When analyzing the limiting deviations and stability of complex engineering systems and dealing with the reliability theory, construction mechanics, and the detection and communication theory, we should know the distribution of the absolute (the largest) maximum of the realization of the stationary random function [4-9]. Such problems as the description of the surface roughness during the mechanical conversion of parts, description of the rough sea surface, seismic and wind actions, analysis of vibrations, etc. are often reduced to the study of characteristics of maximum values. Among the most widely used non-Gaussian random functions, we single out the generalized Rayleigh random process, which can be formulated as [3-5]

$$\eta(t) = \sqrt{[N_1(t) + a_1]^2 + [N_2(t) + a_2]^2}, \quad t \in [0; T].$$

(1)

Here $a_{1,2}$ are some constant values and $N_{1,2}(t)$ are independent, centered Gaussian random processes with variances $\sigma_1^2$ and correlation factors $R(\tau)$. The one- and two-dimensional probability densities $W(x)$ and $W_2(x_1, x_2; \tau)$, respectively, of the process $\eta(t)$ have the form [3-5]

$$W(x) = \frac{x}{\sigma^2} \exp \left[ -\frac{1}{2} \left( \frac{x^2}{\sigma^2} + z^2 \right) \right] I_0 \left( \frac{zx}{\sigma} \right),$$

$$W_2(x_1, x_2; \tau) = \frac{x_1 x_2}{\sigma^4 [1 - R^2(\tau)]} \exp \left\{ -\frac{x_1^2 + x_2^2}{2\sigma^2 [1 - R^2(\tau)]} \right\} \times$$

$$\exp \left[ -\frac{z^2}{1 + R(\tau)} \right] \sum_{n=0}^{\infty} \varepsilon_n I_n \left( \frac{R(\tau)x_1 x_2}{\sigma^2 (1 - R^2(\tau))} \right) I_n \left( \frac{zx_1}{\sigma (1 + R(\tau))} \right) I_n \left( \frac{zx_2}{\sigma (1 + R(\tau))} \right),$$

(2)

where $\{x, x_1, x_2\} \geq 0$, $\varepsilon_n = 1$ for $n = 0$, $\varepsilon_n = 2$ for $n > 0$, $z^2 = (a_1^2 + a_2^2) / \sigma^2$, and $I_n(x)$ is the $n$th-order modified Bessel function. On the basis of Eq. (2), we obtain the expressions for the one-dimensional moments and the correlation of the generalized Rayleigh random process [3-5].

The examples of the process (1) are the envelope of the additive mixture of a harmonic oscillation and narrow-band normal noise [4-6], the output signal of the optimal detector/measurer [7, 8], the distance...
from the origin to the current position of a particle that is wandering randomly in a plane, the absolute value of the particle velocity in a viscous medium [10], etc.

In [7], we obtained the asymptotic expressions

\[ F(h) = P[h_m < h], \quad h_m = \sup\{\eta(t) : t \in [0; T]\}, \quad \eta(t) = \eta(t)/\sigma, \]  

(3)

for the distribution law of the absolute maximum \( h_m \) of the random process (1) for \( z = 0 \) (\( a_1 = a_2 = 0 \)). Using the methods of statistical modeling, we established their applicability range. It is shown that the form (3) of the distribution function \( F(h) \) depends on the analytical properties of the process \( \eta(t) \), in particular, on the existence of a continuous derivative \( \eta(t) \). At the same time, it is of interest to generalize these results to the case \( z \neq 0 \).

The purpose of this paper is to find asymptotic expressions for the distribution law of the absolute maximum of the process (1) for arbitrary \( z \) and to establish their applicability range.

2. DISTRIBUTION OF THE ABSOLUTE MAXIMUM OF THE DIFFERENTIABLE GENERALIZED RANDOM RAYLEIGH PROCESS

Let us consider the generalized random Rayleigh process \( \eta(t) \) (1), which, along with its first derivative \( \dot{\eta}(t) \), is rms-continuous. In this case, according to [9], for the correlation factor \( R(\tau) \) of the quadratures \( N_{1,2}(t) \), the following relation is fulfilled when \( \tau \to 0 \):

\[ R(\tau) = 1 - \frac{\alpha^2 \tau^2}{2} + o(\tau^2), \]  

(4)

where \( o(\tau^2) \) denotes the higher-order infinitesimal terms compared with \( \tau^2 \). Moreover, we assume that, for \( \tau \to \infty \),

\[ R(\tau) = o(\ln^{-1}|\alpha\tau|). \]  

(5)

The quantity \( \alpha \) in Eqs. (4) and (5) has a simple physical meaning. In the time domain, it characterizes the correlation time \( \tau_c \) of the processes \( N_{1,2}(t) \). Indeed, confining ourselves to the parabolic approximation (4) of the function \( R(\tau) \), we determine the correlation time \( \tau_c \) as the duration of the function \( R(\tau) \) at the level 0.5 [7, 8]. Then we have

\[ \tau_c = \frac{2}{\alpha}. \]  

(6)

In the spectral region, the quantity \( \alpha \) describes the equivalent width of the spectral density of the processes \( N_{1,2}(t) \) [3-5]:

\[ \alpha = \left[ \int_{-\infty}^{\infty} \omega^2 G(\omega) \, d\omega / \int_{-\infty}^{\infty} G(\omega) \, d\omega \right]^{1/2}, \]  

(7)

where \( G(\omega) = \sigma^2 \int_{-\infty}^{\infty} R(\tau) \exp(-j\omega \tau) \, d\tau \) is the spectral density of the processes \( N_{1,2}(t) \).

As is obvious from [4, 9], once Eqs. (4) and (5) are fulfilled, the distribution of the number of outliers beyond the level \( h \) of the process (1) is reduced to the Poisson law with increase of \( h \), and for the probability (3) we write

\[ \lim_{h \to \infty} P[h_m < h] = \exp[-\Pi(h)]. \]  

(8)

Here

\[ \Pi(h) = (\xi h/\sqrt{2\pi}) I_0(\xi h) \exp[-(h^2 + z^2)/2] \]  

(9)

is the average number of outliers of the process realization \( \eta(t) \) beyond the \( h \) level over the interval \([0; T]\), and

\[ \xi = T \sqrt{-R''(0)} = T\alpha \]  

(10)

is the referred length of the observation interval, which characterizes the number of independent readings of the quadratures \( N_{1,2}(t) \) and the process \( \eta(t) \) [4].