STATISTICAL PROPERTIES OF THE ILLUMINATED PARTS OF A RANDOM SURFACE

Yu. A. Chernov

We consider the modifications in statistical characteristics caused by shadings in the case of oblique illumination of a random surface. It is shown that shadings and height ruptures in the illuminated part of the surface result in distortions of its height distribution function and peaking of the autocorrelation function for the visible part of the surface. At the limit, this function tends to an exponential one for an arbitrary autocorrelation function of the initial actual surface.

1. INTRODUCTION

In some problems dealing with scattering of waves by a random surface, one must take into account the self-shading of its separate elements due to oblique irradiation.

This issue has been studied by many authors, and a comprehensive review of their results is presented in [1]. However, in the known papers the expressions for the scattered radiation power have been derived in the geometric-optics approximation, with considerable simplifications, which, in particular, resulted in loss of the frequency dependence. Preservation of this characteristic requires a more complete analysis of the scattering, using both the usual height and slope distributions of the surface [1–3] and the autocorrelation function of the visible part of the surface formed by a set of illuminated areas. Considering only the illuminated areas in the presence of shadings results in deformations of the statistical characteristics corresponding to this part of the surface compared with the initial ones: the symmetric distributions become nonsymmetric and the autocorrelation function peaks, since the surface undergoes illumination ruptures.

The known papers dealing with scattering ended in irreconcilable contradictions because of the use of fixed (given) statistical characteristics for the physical surface in question. Despite the genuine mathematical findings in the papers of the last two decades, the neglect of the exponential correlation function prevented progress in theory in terms of closing up calculated and experimental results. The method of bringing theoretical results closer to the actually observed ones consists of determining more accurately the surface characteristics responsible for the formation of the scattering field.

Taking into account shadings and the respective illumination ruptures is the most important difference of this study from the earlier ones.

Below we present one possible variant of obtaining a correction to the initial height distribution of the surface and the autocorrelation function with accuracy acceptable for practical purposes. We also estimated some other statistical parameters describing the illuminated part of the surface (in the model considered here, the nonilluminated part of the surface almost did not participate in the creation of the scattering field). The analysis was done for a stationary surface having an average plane, a finite variance of heights, and an autocorrelation function.

2. DISTRIBUTION OF ELEMENTS OF THE SHADDED PARTS

Assume that the initial surface is normal and has a normal distribution of slopes. We formulate the initial part of the analysis in the form given in [4]. We make use of the expression obtained in [4] for the
probability \( S(\xi_0, p_0, q_0, \alpha, \tau) \) that the point of the surface with derivatives \( p_0 \) and \( q_0 \) in the directions of the \( x \) and \( y \) axes at a height \( \xi_0 \) from the average plane, irradiated at an angle \( \alpha \) to the average plane, will not be shaded by another element of the surface in the interval \( \tau \) on the \( y \) axis toward the source, i.e., the probability that the area that is free of the shading obstacle will not be smaller than \( \tau \) (Fig. 1):

\[
S(\tau, \xi_0) = S(0) \exp \left\{ - \int_0^\tau g(t) \, dt \right\},
\]

where

\[
g(\tau) = \sqrt{\frac{2}{\pi}} \frac{\Lambda(\mu)}{\sigma} \exp \left\{ - \frac{(\xi_0 + \mu \tau)^2}{(2\sigma^2)} \right\},
\]

\[
\Lambda(\mu) = \frac{1}{2} \left[ \sqrt{\frac{2}{\pi}} \frac{\sigma_\alpha}{\mu} \exp \left\{ - \frac{\mu^2}{2\sigma_\alpha^2} \right\} - \text{erfc} \left( \frac{\mu}{\sqrt{2} \sigma_\alpha} \right) \right],
\]

\[
\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt.
\]

Here, \( S(0) \) is equal to unity if \( q_0 < \tan \alpha \) and is equal to zero if the case is different, i.e., \( S(0) = \theta(\mu - q_0) \), where \( \mu = \tan \alpha \), \( \theta(x) \) is the function of a unit jump, and \( \sigma \) and \( \sigma_\alpha \) are the standard deviations of the height and slope of the surface.

The quantity \( S(\tau, \xi_0) \) as a function of \( \tau \) changes from zero to unity: hence, by differentiating \( S(\tau, \xi_0) \) with respect to \( \tau \), one can obtain the distribution density of \( \tau \) for a given level of \( \xi_0 \). Averaging the resulting distribution over \( \tau, \xi_0, p_0 \), and \( q_0 \), we find the mean value \( \tau_{\text{mean}} \) which is required, as is seen below, for determining more accurately the autocorrelation function of a surface with illumination ruptures. We note that averaging over \( p_0 \) gives a factor equal to unity, and averaging over \( q_0 \) gives a factor

\[
G(\mu) = 1 - 0.5 \text{erfc} \left( \frac{\mu}{(\sqrt{2} \sigma_\alpha)} \right) = F(\mu/\sigma_\alpha),
\]

where \( F(x) = (2\pi)^{-1/2} \int_{-\infty}^x e^{-t^2/2} \, dt \).

Upon integration in (1) with allowance for (2) and (3), we have

\[
S(\tau, \xi_0) = G(\mu) \left[ \frac{F(\xi_0/\sigma)}{F((\xi_0 + \mu \tau)/\sigma)} \right]^\Lambda.
\]

Integrating (4) with respect to \( \xi_0 \) and \( \tau \), we find, as in [4], the probability \( S \) of irradiation of surface elements:

\[
S = \frac{1 - \text{erfc} \left( \frac{\mu}{(\sqrt{2} \sigma_\alpha)} \right)/2}{\Lambda + 1} = \frac{F(\mu/\sigma_\alpha)}{\Lambda + 1}.
\]

We note that indeed, as follows from physical considerations, the probability \( S \) does not depend on \( \sigma \), and tends to zero with decrease in \( \alpha \).

Then one must calculate the probability distribution \( W(\tau, \xi_0) \) of lengths of the shaded parts carried by the points of surface with height \( \xi_0 \). Provisionally, we find from (4) the probability of nonshading of these rays on the length \( \tau \):

\[
S_1(\tau, \xi_0) = S(\tau, \xi_0)/G(\mu).
\]