ANALYSIS OF THE ERROR OF A DIRECT ALGORITHM FOR DETERMINING THE DISTANCE TO AN ELECTRIC DIPOLE

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We present an algorithm for combined processing of the vertical and horizontal components of electric and magnetic fields, respectively, which allows us to determine the distance to an equivalent dipole source of electromagnetic radiation. We have found the dependence of mathematical expectation and variance of the distance determination error on the source spectrum and transfer function of the input filter. The estimates for the particular case corresponding to the averaged source of lightning discharges indicate the possibility of using the algorithm under study in systems for locating the lightning centers and allow us to develop the requirements for hardware of such systems. We propose data on the program complex for the system used to locate the lightning centers.

1. INTRODUCTION

In this paper, we discuss a mathematical model of the problem of locating the source of an electromagnetic pulse using the electromagnetic field induced by the latter. The model elements are the arbitrarily oriented electric dipole \( \mathbf{P} = p(t)\mathbf{n}_0\delta(r_0) \) \((p(t)\) is the dipole moment and \( \mathbf{n}_0 \) and \( r_0 \) characterize the dipole orientation and location) located in a halfspace bounded by an infinitely conducting plane and the observation point \( O \) belonging to the above plane. The practical value of this model is the adequate description of the problem of locating thunderstorms for distances from 30 to 150 km. This fact has theoretical grounds \[1\] and was confirmed by numerous practical experiments \[2\]. A more detailed discussion of the adequacy of modeling the problem of locating thunderstorms is beyond the framework of this paper.

As is known \[3\], the electromagnetic field of dipole \( \mathbf{P} \) in vacuum is equal to

\[
\mathbf{E}_0^{(P)} = \frac{1}{4\pi\varepsilon_0} \left\{ \frac{3p}{r^3} + \frac{p'}{cr^2} + \frac{p''}{c^2r} \right\} (\mathbf{n}_0 \cdot \mathbf{e}_0)\mathbf{e}_0 - \left\{ \frac{p}{r^3} + \frac{p'}{cr^2} + \frac{p''}{c^2r} \right\} \mathbf{n}_0,
\]

\[
\mathbf{H}_0^{(P)} = \frac{1}{4\pi} \left[ \frac{p'}{r^2} + \frac{p''}{cr} \right] (\mathbf{n}_0 \times \mathbf{e}_0),
\]

where \( \mathbf{e}_0 \) is the direction from the dipole to the observer, \( \varepsilon_0 \) is the vacuum permittivity, \( c \) is the velocity of light, and \( r \) is the distance from the observer to the dipole.

In the mathematical model used, to determine the field of the dipole \( \mathbf{P} \) in a halfspace bounded by an infinitely conducting plane, it is correct to use the methods of geometric optics. In accordance with these methods, the desired field \((\mathbf{E}^{(P)}, \mathbf{H}^{(P)})\) results from the superposition of the field \((\mathbf{E}_0^{(P)}, \mathbf{H}_0^{(P)})\) and the reflected field \((\mathbf{E}_1^{(P)}, \mathbf{H}_1^{(P)})\), written as the field of the imaginary dipole \( \mathbf{P}^* = p(t)\mathbf{n}_1\delta(r_1) \), which is the mirror reflection of the dipole \( \mathbf{P} \) (see Fig. 1):
Using the Cartesian system of coordinates with origin at the observation point \( O \) and the \( Oz \) axis being the normal of the bounding plane, we present the electric (\( E \)) and magnetic (\( H \)) components of the field in the coordinate form [4, 5]:

\[ E_z(t) = q(t) + q'(t) \frac{v}{\alpha} + q''(t) \frac{u}{\alpha^2} , \]
\[ H_x(t) = \sin \varphi \left( q'(t) \frac{1}{\alpha} + q''(t) \frac{1}{\alpha^2} \right) , \]
\[ H_y(t) = -\cos \varphi \left( q'(t) \frac{1}{\alpha} + q''(t) \frac{1}{\alpha^2} \right) , \]

\[ E_x(t) = E_y(t) = H_z(t) = 0. \]

Here \( \alpha = c/r \) is the inverse of the time of wave propagation from the source to the observer and the variables \( u, v, \varphi, q(t) \) are determined [5] from the equalities

\[ w \sin \varphi = \sin \theta \cos \theta_0 \sin \psi - \cos \theta \sin \theta_0 \sin \psi_0 , \]
\[ w \cos \varphi = \sin \theta \cos \theta_0 \cos \psi - \cos \theta \sin \theta_0 \cos \psi_0 , \]
\[ u = \sin \theta \cos (\varphi - \psi) , \]
\[ v \sin(\psi - \psi_0) = \sin (\varphi - \psi_0) , \]
\[ q(t) = w \alpha^2 p(t) / (2\pi c^2 r) . \]

Other parameters are given in Fig. 1.

From Eqs. (1)–(3) it is obvious that the horizontal and vertical components of the electric and magnetic fields, respectively, at the observation point are presented by different linear combinations of the wave \( q''(t)/\alpha^2 \), induction \( q'(t)/\alpha \), and static \( q(t) \) terms that are normalized to the quantity \( \alpha \) in a different manner. The power-law dependence of the above terms on \( \alpha \) and the linear independence of the system