STRICT ERROR ESTIMATION FOR A SPECTRAL METHOD OF COMPRESSIBLE FLUID FLOW

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ABSTRACT - In this paper, a Fourier Spectral method for compressible flow in n-dimensional space with periodic boundary conditions is constructed. We give a strict error estimation, from which the convergence follows with some assumptions.

1. Introduction

We consider the following compressible fluid equation in n-dimensional space with periodic boundary conditions

$$\partial_t u(1) + (u \cdot \nabla) u(1) - \frac{1}{\rho} \partial_t (K \nabla \cdot u) - \frac{1}{\rho} \sum_{j=1}^{n} \partial_j [\nu \partial_j u(1) + \partial_j u(1)]$$

$$+ \frac{1}{\rho} \partial_t p = f(1) \quad 1 = 1, 2, ..., n,$$

$$\partial_t T + (u \cdot \nabla) T - \frac{1}{\rho T S} (\nabla \cdot u) T - \frac{\nu}{2 \rho T S} \sum_{l,j=1}^{n} (\partial_l u^{(j)} + \partial_j u^{(l)})^2$$

$$- \frac{K}{\rho T S} (\nabla \cdot u)^2 - \frac{\rho S_e}{S_T} (\nabla \cdot u) = 0,$$

$$\partial_t \varrho + (u \cdot \nabla \varrho) + \varrho (\nabla \cdot u) = 0,$$

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where \( \partial_t = \partial / \partial t, \partial_j = \partial / \partial x_j \), \( u \) is the velocity vector and \( u = (u^{(1)}, u^{(2)}, ..., u^{(n)}) \), \( T \) is the absolute temperature, \( \nu(T, \varphi) \) is the viscosity, \( K(T, \varphi) = \nu'(T, \varphi) - 2/3 \nu(T, \varphi) \) where \( \nu' \) is the second viscosity, \( \mu(T, \varphi) \) is the heat conduction coefficient, and \( S(T, \varphi) \) is the entropy, \( S_T = \partial S / \partial T \) and \( S_{\varphi} = \partial S / \partial \varphi \). Assume that the solution of the equation has period \( 2\pi \) for each space variable \( x_j \) and the equation is solved on the domain \( (x,t) \in Q \times (0, t_0] \), where
\[
Q = \{ x = (x_1, x_2, ..., x_n) \mid -\pi \leq x_j \leq \pi, \quad j = 1, 2, ..., n \}.
\]

Many papers have been devoted to the numerical solution of this problem (see [1-2]). Guo Ben-yu proposed a technique to prove the strict error estimation of difference methods to solve this problem with periodic boundary conditions and also the problem with the first kind boundary condition (see [3, 4]). He obtained the convergence. In the proof of convergence in [4], we require that the approximation error is \( O(h^n) \) where \( h \) is the mesh spacing for the space variables. Because the accuracy of approximation of the difference scheme of [3-4] is of order \( O(h^2) \), even if the solution of P.D.E is sufficiently smooth, the convergence is only for the problem in two dimensional space. For spectral methods, however, the order of accuracy of approximation could be arbitrarily high, provided the solution of P.D.E. is sufficiently smooth. This makes it possible to consider the convergence of the problem in n-dimensional space. On the other hand, we can obtain more refined estimations in some cases with periodic conditions.

In this paper, we discuss a Fourier spectral method for this problem. According to the idea in [3-4], we assume \( P = R_0 T \) where \( R \) is a constant. Let \( \varphi = \ln \varphi \). Then the original equation is changed into
\[
\partial_t u^{(l)} + (u \cdot \nabla) u^{(l)} - e^{-\varphi} \partial_l (K \nabla \cdot u) - e^{-\varphi} \sum_{j=1}^{n} \partial_j \left[ \nu(\partial_j u^{(l)}) + \partial_l u^{(j)} \right] + R \partial_t T + RT \partial_t \varphi = f^{(l)}, \quad l = 1, 2, ..., n,
\]
\[
\partial_t T + (u \cdot \nabla) T - e^{-\varphi} T^{-1} S_T^{-1} (\nabla \mu \nabla) T - 1/2 \nu e^{-\varphi} T^{-1} S_T^{-1}
\]
\[
\sum_{l,j=1}^{n} (\partial_l u^{(j)} + \partial_j u^{(l)})^2
\]
\[
- K e^{-\varphi} T^{-1} S_T^{-1} (\nabla \cdot u)^2 - S_{\varphi} S_T^{-1} (\nabla \cdot u) = 0,
\]
\[
\partial_t \varphi + (u \cdot \nabla) \varphi + \nabla \cdot u = 0.
\]