A SYSTEM FOR THE AUTOMATIC TRIANGULATION OF REGIONS CONTAINING CUTS WITH ELEMENTS HAVING SIX NODES

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In this article, we consider the problem of constructing an automatic system for partitioning two-dimensional simply or multiply connected multicomponent regions containing cuts into triangular finite elements with six nodes. Minimization of the strip of nonzero elements in the matrix involved in the finite-element method is performed by the Cathill-Mackey algorithm. The effectiveness of the system has been checked with a series of test examples. Bibliography: 5 titles.

When the finite-element method (FEM) is used, one of the central problems is to divide the region into finite elements. Most often, triangles are chosen to be the finite elements. Brebbia and Konnor [1], George and Liu [2], and Segerlind [3], Sergienko, Skopetskii, and Deineka [4] have devoted particular attention to the problem of triangulation of regions. In the system for the automation of calculation of fields and construction optimization (SARPOK) [4], developed at the Institute of Cybernetics of the National Academy of Sciences of Ukraine, the triangulation of regions and node numbering is carried out with the use of the system PAROR [5] in the case of piecewise-linear functions used in FEM.

However, solving problems of moisture- and moisture-mass-transport in an aeration zone by using the finite-element method, we get systems of algebraic equations with badly conditioned matrices. Their condition numbers satisfy the estimate

$$\text{cond}(A) \leq \frac{k_{\max}}{k_{\min}} \cdot \frac{1}{h^2},$$

Here $k_{\min} \to 0$ as $\theta \to 0$, $\theta$ is the viscosity of the soil, $\bar{h}$ is the smallest characteristic length of triangles that make the partition of the region, and $k = k(\theta)$ is the moisture-transport coefficient. On the other hand, obtaining a more exact solution by making refinement of finite elements leads, in turn, to the fact that $\bar{h} \to 0$. These two circumstances make it difficult to obtain sufficiently exact approximate solutions, even by using modern computers, if the class of piecewise-linear functions are used in the finite-element method.

Polynomials of higher degrees in the finite-element method allow one to construct numerical schemes with a higher precision. In this case, the use of a coarser grid and, for example, piecewise-quadratic functions satisfying estimate (1) allows one to obtain numerically a sufficiently exact approximation of a badly conditioned moisture-transport problem.

Below we describe the automatic system SART2 for partitioning multicomponent two-dimensional regions having cuts (which include multiply connected regions) with triangular finite elements containing six nodes each. Using an approach to describe the geometry of the region and its digitization which is implemented in the system SARPOK, we develop here the main principles given in [2, 5] and apply them to schemes of the finite-element method with an increased (second-order) degree of approximation. For this scheme, software was developed for triangulating multicomponent regions with cuts that have an arbitrary orientation. This software is called SART2 (the system for automatic partitioning multicomponent regions with cuts for the case of classes of piecewise-quadratic functions, possibly with jumps, in the finite-element method). A number of numerical experiments have been carried out to compare partitioning characteristics of schemes with first and second degree of approximation.

The discretization of any domain is split into two steps:

1. partitioning the body into finite elements (triangles in this article);
2. indexing the elements and partition nodes.


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To triangulate a multicomponent region, including regions with cuts, first we input the information about the region into the computer. This can be done by using a graphical interface. To make use of the interface, the user must first represent the region as a union of quadrangular subregions, called zones. The borders should separate zones where there are changes in geometry, in the load applied, or in the physical properties of the region. An example of such a partition, called OBL2, is given in Fig. 1. Here thick lines denote fine inclusions. In the middle of the region there is a cavity. Representing the geometry of the region in such a way, the zones form horizontal and vertical layers.

The quadrangular zones are partitioned into quadrangular elements by connecting the nodes at the opposite sides of each zone with line segments. In so doing, the number of nodes at opposite sides of each zone must be the same; moreover, the zones that have a common boundary must also have the same number of nodes, and the relative position of the nodes should coincide.

In this article, we partition each side of the zones with a uniform grid with a given number of nodes for every horizontal and vertical layer of the zones. The intersections of the lines determine interior nodes, which are vertices of triangles. The obtained quadrangular elements are subdivided into triangles by drawing the short diagonal in every quadrangular element. After that, nodes are added at the midpoints of the sides of every triangle of the partition. As a result, we obtain a triangulation of the region and a set of partition nodes for piecewise-quadratic basic functions used in the finite-element method such that there are six nodes for every triangle: three nodes are vertices of the triangle and three nodes lie in the midpoints of its sides.

It is necessary to index the set of partition nodes. Since the routine that divides the region consecutively covers all of the zones in a cycle, first it triangulates a zone and then indexes its nodes. The indexing increases from left to right and from bottom to top, covering all nodes of every zone, with the starting index equal to the last index used for the previous zone increased by one. In so doing, it is also necessary to take into account that some nodes have already been indexed in a previous cycle and should not be indexed again. An exception is made for border nodes of the zones that belong to inclusions: they have two indices, i.e., the nodes with the same coordinates have different indices for different sides of the inclusion.

As a result of such a partition, we obtain three files:

1. file FCN containing information about triangles of the region partitioning and having the following structure: the zone index, the index and corresponding coordinates of vertices of the triangle, indices and corresponding coordinates of midpoints of the sides of the triangle.
2. file FCK containing information about segments that split the boundary of the region; it has the