AN ANALYTICAL STUDY OF THE SENSITIVITY OF A DISCRETE STOCHASTIC MODEL FOR SALES DYNAMICS

V. D. Romanenko, V. N. Podladchikov, and A. S. Kopychko

We consider a discrete model for sales dynamics in the case of a stochastic model of the market. The model includes “fast” and “slow” components of the market situation described by a stochastic process of “white noise” type and the correlated stochastic process. By using an integral representation of the main characteristics of the Kalman filter, we obtain expressions for stochastic parameters of additional errors of the estimate that arise in the case where the characteristics of noises are inexact. We make an asymptotical analysis of these expressions and give recommendations for the price-forming strategy in the case of uncertainty of the market situation. Bibliography: 2 titles.

INTRODUCTION

A model for sales dynamics was obtained in [1] in the case of continuous time. This model was suggested for the synthesis of decision-making algorithms for the price-forming strategy at the stage of sales.

The suggested model allows one to develop an optimal price-forming strategy based on contradictory motives: on the one hand, the aim is to maximize the income and, on the other hand, to minimize the stock.

The seller uncertainty in the market situation is modeled by introducing Markov stochastic processes.

The change in the amount of goods sold as a function of time is described by differential equations that relate the amount of goods sold at every moment to the entire history of the pricing and demand fluctuations.

As the information about the goods sold comes into the demand estimate block, the current value of the demand parameters is estimated and, on this basis, the new price is calculated.

The quality of the price-forming strategy depends on the precision of the market-demand estimate, which, in turn, is determined by the precision in determining the model parameters.

For practical use of this model, one needs to make an analysis of the quality of the estimates for different conditions of inexact values of the actual parameters. In [1], a one-dimensional system was modeled on a computer for testing the dependence of the accuracy of the estimate on errors in statistical parameter data. It was shown that the quality of the price formation is sensitive to errors in determining the parameters.

In [2] an analytical approach was used. It allows one to study general properties of the algorithm for estimating the market situation with an a priori uncertainty and to find conditions under which the estimates allow the decision-making person to reliably control the price-forming strategy.

The estimate of the market situation, in the considered model, is determined from the sales-dynamics data that, in practice, come at discrete periods of time.

Hence, it is of practical interest to develop a discrete model for the sales dynamics and to analyze its quality in the conditions of an a priori uncertainty. The analysis of the sensitivity of the proposed sales-dynamics model is given by using an analytical representation of the Kalman filter as an explicit function of the parameters of the discrete stochastic model.

THE SETTING OF THE PROBLEM

We assume that the demand for certain goods changes in a random way described by a “slow” component of the demand, which is determined by the market condition and a “fast” component related to local fluctuations of the market at the location of the sales.
The equation of the sales dynamics has the form

\[ S_k = S_{k-1} + f(c_k)(t_k - t_{k-1}) + H\eta_k + W_k. \]  

(1)

Here \( S_k \) is the quantity of goods sold up to the time \( t_k \), \( H\eta_k \) is the "slow" component of the demand change, where \( H \) is a matrix of dimension \([1 \times n]\), and \( \eta_k \) is described by the equation of the forming filter:

\[ \eta_k = B\eta_{k-1} + V_k, \]  

(2)

\[ E[\eta_0] = \eta(0), \]

\[ E[(\eta_k - \eta(0))^2] = P_0, \]  

(3)

\( f(c) \) is a monotone decreasing price function having an inverse, \( B \) is an \([n \times n]\)-dimensional matrix characterizing the influence of the price on the rate of sales, \( V_k \) is an independent random sequence with zero mean and covariance matrix \( Q \), and \( W_k \) is an independent random sequence characterizing the "fast" change of the market with zero mean and variance \( R \).

The information on the market situation can be made more precise by using the information on the unsold stock, defined by the quantity \( \Delta S = S_0 - S_k \).

In so doing, we should minimize the influence of local fluctuations which do not influence the general tendencies in the demand change. The model of the system that characterizes the condition and change of the "slow" component of the market condition, \( \eta_k \), can be described in terms of the space of states of Eqs. (1), (2).

To estimate the vector \( \eta_k \), we use the procedure of Kalman filtration.

The algorithm for estimating the vector \( \eta_k \) includes the following stages.

1. **Filtration.**

\[ \hat{\eta}_{k,k} = \hat{\eta}_{k,k-1} - K_k(\Delta S_k - \Delta S_{k-1} + f(c_k)(t_k - t_{k-1}) + H\hat{\eta}_{k,k}), \]  

(4)

\[ P_{k,k} = (I - K_kH)P_{k,k-1}. \]  

(5)

Here \( \hat{\eta}_{k,k} \) is an estimate of the vector \( \eta_k \) at the time \( t_k \) from the experimental data

\[ \Delta S_1 - \Delta S_0, \quad \Delta S_2 - \Delta S_1, \quad \ldots, \quad \Delta S_k - \Delta S_{k-1}; \]

\[ K_k = P_{k,k-1}H^T(HP_{k,k-1}H^T + R)^{-1} \]  

(6)

is the filter gain coefficient; \( P_{k,k} \) is the covariance matrix of the estimate error \( \hat{\eta}_{k,k} \) of the vector \( \eta_k \).

2. **Extrapolation.**

\[ \hat{\eta}_{k+1,k} = B\hat{\eta}_{k,k}, \]  

(7)

\[ P_{k+1,k} = BP_{k+1,k}B^T + Q. \]  

(8)

The quality of the price-forming policy depends on the accuracy of the estimate of the vectors \( \eta_k \) which, in turn, is determined by the actual variances of the random sequences \( V_k \) and \( W_k \) and also by the errors of the calculated values of \( Q \) and \( R \) used in the construction of the algorithm for estimating \( \eta_k \).

In this article, we consider the problem of the analytical study of the sensitivity of the algorithm for estimating the dependence of the market demand on errors in determining the noise characteristics in model (1)–(3) by using a representation for the actual value of the covariance matrix \( P_{k,k} \) in terms of an explicit function of these errors in the case of a one-dimensional sales model (\( H \) and \( B \) are coefficients).