EVOLUTION OF LINEAR WAVES IN A LIQUID IN THE PRESENCE OF A CURTAIN OF BUBBLES

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We investigate certain features of the evolution of waves in an acoustically compressible liquid in passage through a curtain of bubbles between two parallel planes. We consider the problem of reflection from a plane solid wall separated from the liquid by a curtain of bubbles. Investigations showed that in relation to the duration of the pulse it is possible to select a curtain with corresponding parameters to moderate the effect of the wave on the wall.

The introduction of a small amount of gas into a liquid in the form of gas bubbles distributed throughout the volume of the liquid substantially changes the acoustic properties of the liquid [1-4]. In particular, an anomalous decrease in the rate of propagation of sonic perturbations and substantial enhancement of the dissipation mechanisms occur. All this makes it possible to use bubble curtains to suppress pressure pulses in a liquid. In [4, 5] certain special features of the evolution of nonlinear waves in a liquid and a gas in passage through a curtain of bubbles was studied and the effect of a shock wave on a solid wall in the presence of a curtain of bubbles with a variable gas content was investigated. In the present work we investigate certain features of the evolution of waves in an acoustically compressible liquid in passage through a curtain of bubbles in the region between two parallel planes. Moreover, we consider the problem of reflection of waves from a plane solid wall separated from the liquid by a curtain of bubbles.

1. Basic Equations. Let the curtain be a mixture of a liquid with spherical gas bubbles of identical radius. Here the wavelength is much larger than the width of the curtain of bubbles. This makes it possible to assume that the distributions of the perturbations of the parameters (radius, volumetric content of bubbles, pressure in the liquid) are homogeneous over the thickness of the curtain. Let us consider a certain cylindrical volume of bubble liquid of unit cross section between planes that are the boundaries of the curtain. Then for the mass of the liquid and the number of bubbles in this volume in the absence of phase changes and fragmentation and flocculation of the bubbles we may write

\[
\rho_\text{liq}^0 (1 - \alpha_g) l = m = \text{const}, \quad n l = N = \text{const}, \quad \alpha_g = \frac{4}{3} \pi a^3 n. \tag{1}
\]

From Eqs. (1) in a linearized approximation we may obtain

\[
(1 - \alpha_g) \rho_\text{liq}^0 - \frac{3 \rho_\text{liq}^0 \alpha_g}{a_0} a + \frac{\rho_\text{liq}^0}{l_0} l = 0. \tag{2}
\]

To describe radial motions of the bubbles in the curtain we take the Rayleigh–Lamb equation, which in the case of polytropic behavior of the gas in the bubbles in a linearized approximation is presented in the form

\[
\frac{d^2 a}{dt^2} + 2\beta \frac{da}{dt} + \omega_R^2 a = -\frac{p_\text{liq}}{\rho_\text{liq}^0 a_0}, \tag{3}
\]
\[ \beta = \frac{2\nu}{\alpha_0}, \quad \omega_R = \sqrt{\frac{3\gamma p_0}{\rho_{\text{liq}0}} \alpha_0^{-1}} \]

The equation of state of the liquid will be taken in the acoustic approximation:

\[ p_{\text{liq}} = C_{\text{liq}}^2 \rho_{\text{liq}}. \] \( \text{(4)} \)

Let us consider the evolution of a plane one-dimensional wave in the liquid interacting with the curtain of bubbles. By virtue of the assumption that the wavelength is much larger than the bubble-curtain thickness, in what follows we will take the curtain as a certain reflecting surface that intersects the axis \( Ox \) at \( x = 0 \). Outside this surface \( (|x| > 0) \) the following equations of continuity and pulses are valid for perturbations:

\[ \frac{\partial p_{\text{liq}}}{\partial t} + \rho_{\text{liq}0} \frac{\partial v}{\partial x} = 0, \quad \rho_{\text{liq}0} \frac{\partial v}{\partial t} = -\frac{\partial p_{\text{liq}}}{\partial x}. \] \( \text{(5)} \)

We will assume that the initial perturbation is a wave that propagates in the part to the left of the reflecting surface \( (x < 0) \). When the wave interacts with the bubble curtain, we assume equality of pressures over its two boundaries. Then this condition of continuity of pressure on the reflecting surface can be written in the form

\[ p^{(0)}(0) + p^{(t)}(0) = p^{(t)} = p_{\text{liq}}, \quad x = 0. \] \( \text{(6)} \)

We will assume that the velocity of the liquid suffers a discontinuity on the reflecting surface, and its magnitude is equal to the rate of change of the bubble-curtain thickness:

\[ v^{(0)} - (v^{(0)} - v^{(t)}) = \frac{dl}{dt}, \quad x = 0. \] \( \text{(7)} \)

In the case where a perfectly rigid wall is located behind the curtain, we assume that \( \delta^{(0)} = 0 \), with \( p^{(t)} \) expressing the pressure experienced by the wall.

2. Dispersion Analysis. Suppose a plane harmonic wave is incident on a reflecting surface. Then the motion in the part to the left of the reflecting surface \( (x < 0) \) is a superposition of two waves: an incident one

\[ p^{(0)} = A_p^{(0)} \exp i (kx - \omega t), \quad v^{(0)} = A_v^{(0)} \exp i (kx - \omega t) \] \( \text{(8)} \)

and a reflected one

\[ p^{(r)} = A_p^{(r)} \exp i (-kx - \omega t), \quad v^{(r)} = A_v^{(r)} \exp i (-kx - \omega t), \] \( \text{(9)} \)

and the part to the right of the reflecting surface \( (x > 0) \) there is only one, transmitted wave:

\[ p^{(t)} = A_p^{(t)} \exp i (kx - \omega t), \quad v^{(t)} = A_v^{(t)} \exp i (kx - \omega t), \] \( \text{(10)} \)

where \( k = \omega/C_{\text{liq}} \) is the wave number. Here, on the basis of Eqs. (5) the amplitudes of the pressures and the velocity are interrelated as

\[ A_p^{(0)} = \rho_{\text{liq}0} C_{\text{liq}} A_v^{(0)}, \quad A_p^{(r)} = -\rho_{\text{liq}0} C_{\text{liq}} A_v^{(r)}, \quad A_p^{(t)} = \rho_{\text{liq}0} C_{\text{liq}} A_v^{(t)}. \] \( \text{(11)} \)

The interrelation between the three (incident, reflected, and transmitted) waves is determined by boundary conditions (6) and (7) on the reflecting surface.

In the case of harmonic waves the perturbations of the parameters in the curtain are represented in the form