MODELING OF HEAT AND MASS TRANSFER IN CAPILLARY-POREOUS MATERIALS

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A system of differential equations for heat and mass transfer in capillary-porous materials in a hygroscopic moisture-content field is derived with allowance for kinetics of moisture desorption, two-phase filtration, and liquid pressure determined by the action of surface forces. An analysis is made of the results of a computational experiment illustrating evolution of the moisture content, temperature, and vapor pressure fields against the material characteristics and kinetic coefficient.

Introduction. Heat and mass transfer in capillary-porous media has attracted the attention of many researchers owing to the considerable scientific and practical importance of these processes.

To describe moisture transfer in capillary-porous bodies, A. V. Luikov has introduced the unified potential \( \theta_{m,t} \) by analogy with the thermal potential (temperature) [1, 2]. To construct a scale of the potential \( \theta_{m,t} \) and determine the moisture content as a function of \( \theta_{m,t} \), a potentiometric method has been developed, which consists in measuring the specific moisture content of an investigated capillary-porous body as a function of the moisture content of the reference body, both being in direct contact in the state of thermodynamical equilibrium. The dependences \( u = f(\theta_{m,t}) \) are rather different in form (concave, convex, flat, etc.).

In [3], it is shown with the aid of the method of measuring the equilibrium curves \( u = f(u_{ref}) \) (\( u_{ref} \) is the specific moisture content of the reference body) and the method of mercury porosimetry that the function of pore distribution with respect to capillary pressure is the governing factor of moisture distribution in a porous body or in a system of contacting porous bodies that are in the equilibrium state. An analysis of experimental studies of the mechanisms of mass transfer in the drying of the model [4] and real materials, including those with the use of radioactive tracers [5], made by these authors testifies to the decisive influence of capillary and surface forces on mass transfer and the drying rate.

In some works [6-9], the process of mass transfer in drying has been accounted for by driving forces determined by pressure gradients of the vapor and liquid (or capillary pressure) with use of structural characteristics of capillary-porous materials.

Mathematical Model. We will consider a nondeformable capillary-porous body. The investigated system consists of a skeleton of the capillary-porous body, liquid and moist air representing a binary vapor-gas mixture in the body capillaries.

The heat-transfer equation can be written in the form

\[
[(1 - m) c_s \rho_s + a_v \rho_m c_m + u \rho_s c_{liq}] \frac{\partial T}{\partial t} = \text{div}(\kappa_{el} \nabla T) - c_m \rho_m V_m \nabla T - c_{liq} \rho_{liq} V_{liq} \nabla T + LI.
\]  

The first term on the right-hand side of the equation takes into account heat transfer by heat conduction, the second and third terms — heat transfer due to molar transfer of a liquid and a vapor-gas mixture, the fourth term — phase changes. It has been assumed that the vapor in capillaries is in thermal equilibrium with the liquid [6].

In [1], a system of equations is proposed for heat and mass transfer in capillary-porous media involving phase changes. However, the system is not closed since the introduced criterion of a phase transition in the source term is not determined. The system of equations obtained in [9] is closed with the aid of the sorption (desorption)
isotherm and the hypothesis on an equilibrium regime (momentarily reaching the equilibrium state). However, this approach does not allow for the dynamics of a phase change (a nonequilibrium regime).

Let the sorption (desorption) isotherm $u_e = f(\varphi, T)$, where $\varphi = P_v / P_{sat}$, $P_v$ is the partial vapor pressure (Pa), be known from experimental studies.

Note that when the pressure of the medium considerably differs from atmospheric pressure, it is necessary to know the isotherm as a function of the medium pressure $u_e = f(\varphi, T, P_m)$. Then the equation for vapor sorption (desorption) kinetics is

$$ I = \frac{db}{dt} = \gamma (\varphi - \varphi_e) = \frac{\gamma}{P_{sat}} (P_v - P_e), \quad (2) $$

where $P_e$ is the equilibrium partial vapor pressure (Pa) determined from the isotherm.

In the case of the equilibrium sorption (desorption) conditions, $a = u_e \rho_s$; consequently,

$$ I = \frac{db}{dt} = \frac{d \rho_u u_e (\varphi, T)}{dt}. \quad (3) $$

When the moisture content $u$ of the body is higher than the maximum hygroscopic one, then the equilibrium vapor pressure approaches the saturated vapor pressure $P_e = P_{sat}$ and Eq. (2), in some approximation, can be written as

$$ I = \frac{db}{dt} = \gamma (\varphi - 1). \quad (4) $$

In this case, to determine the kinetic coefficient, it is also necessary to carry out experimental studies.

To evaluate the specific heat of moisture evaporation of the material, we use the Clapeyron–Clausius equation

$$ L = \frac{RT^2}{\nu_v} \left( \ln P_v \right) . \quad (5) $$

The vapor pressure $P_v$ can be expressed from the equation of the desorption isotherm $P_v = F(u_e, T)$; then

$$ L = \frac{RT^2}{\nu_v} \left( \ln F (u_e, T) \right) . \quad (6) $$

This expression takes into account the amount of heat needed in drying to overcome the binding energy of the moisture with the material.

The continuity equations for a liquid and a vapor-gas mixture are written in the form

$$ \frac{\partial (m \rho_{liq} \theta_{liq})}{\partial t} = - \nabla j_{liq} + I_1 . \quad (7) $$

$$ \frac{\partial (m \rho_m \theta_m)}{\partial t} = - \nabla j_m + I_2 . \quad (8) $$

The motion of phases in a porous material under evaporation conditions is determined by the equations of two-phase filtration complicated by vapor diffusion in the gas phase:

$$ V_{liq} = m \theta_{liq} \nu_{liq} = - \frac{k_f \mu_{liq}}{\theta_{liq}} \nabla p_{liq} , \quad (9) $$