ANALYTICAL METHODS OF INVESTIGATION OF THE THERMAL STATE OF A REGION WITH A MOVING BOUNDARY UNDER THE CONDITIONS OF NONSTATIONARY HEAT TRANSFER TO THE EXTERNAL MEDIUM

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An analytical method of solution of combined problems of nonstationary heat conduction for a region with a boundary moving according to a known law and with a time-variable coefficient of heat transfer is developed. The idea of splitting the kernel of the obtained generalization of a singular integral Fourier transform with respect to a space variable provides a basis for the method. Theoretical results are used in mathematical simulation of heat transfer processes in the region with a moving boundary under the conditions of nonstationary heat transfer to the external medium.

The combined problem of nonstationary heat conduction for a region with a boundary moving according to a known law occupies a special place in mathematical theory of heat conduction [1]. The necessity of allowing for the mobility of the boundary arises, in particular, in mathematical simulation of high-temperature modes of the effect which are accompanied, for example, by destruction or ablation melting of surface layers of material [2]. Physical realization of the mentioned models of thermal effect inevitably leads to changes in the conditions of heat transfer to the external medium and manifests itself in a time variation in the coefficient of heat transfer.

The mathematical model of the heat transfer process in the region with a boundary uniformly moving according to the law \( v = 2 \beta \Phi \) has the form

\[
\frac{\partial \theta}{\partial \Phi} = \frac{\partial^2 \theta}{\partial \xi^2}, \quad \xi > v(\Phi), \quad \Phi > 0,
\]

\[
\theta (\xi, 0) = 0,
\]

\[
\left. \frac{\partial \theta (\xi, \Phi)}{\partial \xi} \right|_{\xi = v(\Phi)} = Bi(\Phi) \left| \theta (\xi, \Phi) \right|_{\xi = v(\Phi)} - \zeta(\Phi),
\]

where

\[
\xi = \frac{x}{x_a}; \quad \Phi = \frac{\nu}{x_a}; \quad \theta = \frac{T - T_0}{T_{\infty0} - T_0}; \quad \zeta = \frac{T_c - T_0}{T_{cd0} - T_0}; \quad Bi = \frac{\alpha}{\lambda}.
\]

In accordance with the meaning of the problem solved, the functions Bi(\( \Phi \)), \( \zeta(\Phi) \), and \( v(\Phi) \) are non-negative, and the functions Bi(\( \Phi \)) and \( \zeta(\Phi) \) are absolutely integrable on the half-open interval \([0, +\infty)\) when \( \Phi \geq 0 \).

We emphasize that determination of an analytical solution of problem (1) involves fundamental difficulties. This is basically caused by the dependence of the function Bi on the Fourier number \( \Phi \). This problem
is discussed in [1]. It should, however, be noted that in the case of absolutely integrable functions $B_i(F_0)$ and $\zeta(F_0)$ when $F_0 \geq 0$, the conditions of the theorem [3] of existence and uniqueness of the problem considered are met, i.e., the solution $\theta(\xi, F_0) \in L^2(\nu(F_0), +\infty) L^1[0, +\infty)$ of problem (1) exists and is unique. The present study is aimed at obtaining this solution.

To simplify further consideration, we pass over to a moving system of coordinates using a new space variable

$$X = \xi - v(F_0).$$

In this case, problem (1) acquires the following form:

$$\frac{\partial \theta}{\partial F_0} = \frac{\partial^2 \theta}{\partial X^2} + 2\beta \frac{\partial \theta}{\partial X}, \quad X > 0, \quad F_0 > 0,$$

$$\theta(X, 0) = 0,$$

$$\left. \frac{\partial \theta(X, F_0)}{\partial X} \right|_{X=0} = B_i(F_0) \left[ \theta(X, F_0) \right|_{X=0} - \zeta(F_0).$$

The solution of problem (2) is based on the use of a singular integral transform with respect to a space variable $X$:

$$u(\lambda, F_0) = \Phi[\theta(X, F_0)] = \int \theta(X, F_0) \rho(X, F_0) K(X, \lambda, F_0) dX =$$

$$= \int_0^\infty \theta(X, F_0) \exp(\beta X) \left\{ \cos(\lambda X) + \frac{h(F_0)}{\lambda} \sin(\lambda X) \right\} dX,$$

$$\theta(X, F_0) = \Phi^{-1}[u(\lambda, F_0)] = \frac{2}{\pi} \int_0^\infty u(\lambda, F_0) \exp(-\beta X) \times$$

$$\times \left\{ \cos(\lambda X) + \frac{h(F_0)}{\lambda} \sin(\lambda X) \right\} \frac{\lambda^2 d\lambda}{\lambda^2 + h^2(F_0)};$$

$$h(F_0) = B_i(F_0) + \beta.$$

Expression (3) is a generalization of the combined integral Fourier transform [4, 5].

Direct use of (3) for obtaining the solution of problem (2) is impossible [6] because its kernel $K(X, \lambda, F_0)$ depends not only on the space variable $X$ and the parameter of integral transformation $\lambda$, but also on the Fourier number $F_0$. This, in particular, leads to the fact that

$$\Phi \left[ \frac{\partial \theta(X, F_0)}{\partial F_0} \right] \neq \frac{\partial u(\lambda, F_0)}{\partial F_0}.$$

To overcome these difficulties, we use the Euler formulas and transform the kernel of integral transform (3):