COMPARATIVE ANALYSIS OF TURBULENCE MODELS

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A comparative analysis is performed for a complete locally anisotropic turbulence model of the second order and existing turbulence models. The comparison draws on experimental data, data of a direct numerical simulation of the nonstationary Navier–Stokes equations for a developed channel flow and a uniform channel flow with a constant velocity shift, and results for turbulence damping behind a grid. The K–ε model and the quasi-isotropic turbulence model are shown to have marked disadvantages, especially in describing turbulent flows with a high degree of anisotropy of pulsatory motion. Use of a locally anisotropic turbulence model improves the accuracy of determining Reynolds stresses. Consideration is given to the advantages and disadvantages of the turbulence models discussed.

Introduction. Applied problems of hydrodynamics and heat and mass transfer in turbulent fluid flows must be solved using one or another turbulence model. A great many works have now been published that describe turbulence models of different degrees of complexity. The K–ε turbulence model is the most popular. However, in many cases use of the K–ε model for describing intricate turbulent flows does not produce results that agree with experimental data. Therefore, it is reasonable to employ a more complex turbulence model for Reynolds stresses that takes account of the anisotropy of the latter. Comparison of numerical results with reliable experimental data has revealed that the turbulence models for Reynolds stresses that have been developed to date describe the dissipation process and the process of energy redistribution between the components of the Reynolds stresses as a result of pressure pulsations with insufficient accuracy. In recent years, turbulence models have been actively worked out that take into account not only the anisotropy of the Reynolds stresses but also the anisotropy of the dissipation processes.

The current work aims at analyzing the accuracy and ranges of applicability of the above turbulence models. Consideration is given to some existing turbulence models and a complete turbulence model of the second order obtained in [8-10]. A comparison is made using experimental data for a uniform channel flow with a constant velocity shift and data for turbulence damping behind a grid. Below, a brief description of the principal turbulence models discussed in the work is given.

K–ε Model. The published literature has offered a great number of various versions of the K–ε model. A review of works published before 1984 is presented in [1]. The basic equations of the model have the form

\[
\frac{DK}{Dt} = P_k - \bar{\varepsilon} + D_k;
\]

\[
\frac{De}{Dt} = C_{e1} f_1 \frac{\varepsilon P_k}{K} - C_{e2} f_2 \frac{\varepsilon^2 K}{K} + C_{e3} \left( \frac{K R_k \varepsilon_d}{\varepsilon} \right) \frac{\varepsilon}{K} + E;
\]

\[
b_{ij} = \frac{R_{ij}}{2K} - \frac{1}{3} \delta_{ij} = -\nu_{roman} \frac{1}{K} S_{ij}, \quad S_{ij} = \frac{1}{2} (U_{i,j} + U_{j,i});
\]
\[ v_t = C_\mu f_\mu \frac{K^2}{\varepsilon}, \quad \bar{\varepsilon} = \varepsilon - D; \]  

\[ C_\mu = 0.09, \quad C_{e1} = 1.45, \quad C_{e2} = 1.9, \quad C_\varepsilon = 0.15. \]  

The various versions of the model differ in the form of the empirical wall functions \( f_\mu, f_1, f_2, E, \text{ and } D \). A table that provides corresponding relations for these functions proposed in published works is given in [2]. It should be noted that the introduction of such a large number of wall functions indicates serious shortcomings of the model that make it impossible to calculate boundary-layer flows at the wall with an acceptable degree of accuracy. An analysis [3] demonstrated that the \( K-\varepsilon \) model can be derived from a more complex model for Reynolds stresses under the following assumptions: small-scale motions are isotropic, velocity gradients are small, and equilibrium between the generation of turbulent energy and the dissipation rate is maintained in the flow. Furthermore, the model is valid only for plane flows with a simple velocity shift. The above constraints narrow the range of applicability of the considered model so much that it is difficult in practice to find a flow for which the \( K-\varepsilon \) model can be used. However, the latter is employed in many works for calculating stratified, swirling, and other intricate flows. As a rule, in these cases, in order that calculated results agree with experimental data, various empirical functions are introduced that do not have sufficient physical substantiation.

**Quasi-isotropic Turbulence Model for Reynolds Stresses.** Among contemporary turbulence models, turbulence models of the second order are widely popular. A way to construct them was proposed in the early 70s in [4]. The essence of this approach lies in the fact that the Reynolds stresses should be determined from an exact equation for second single-point moments. Results of the latest investigations are published in [5]. The basic equations of the model have the form

\[ \frac{D R_{ij}}{Dt} = F_{ij} + P_{ij} + \Phi_{ij} - 2\varepsilon_{ij} + D_{ij}, \]  

\[ \frac{D\varepsilon}{Dt} = C_{e1} \frac{\varepsilon P_k}{K} - C_{e2} \frac{\varepsilon^2}{K} + C_\varepsilon \left( \frac{K}{\varepsilon} R_{ij} \varepsilon_{ij} \right), \]  

\[ \varepsilon_{ij} = \frac{1}{3} \varepsilon \delta_{ij}, \quad D_{ij} = -C_s \left[ \frac{K}{\varepsilon} R_{ij}\right]_{,ij}, \]  

\[ \Phi_{ij} = -C_1 \varepsilon b_{ij} + \]  

\[ + 4K \left[ d_1 S_{ij} + d_2 \left( b_{ij} S_j^p + b_{ji} S_i^p \right) - \frac{2}{3} \left( bS \right) \delta_{ij} + d_3 \left( b_{ij} W_{pj} + b_{ji} W_{pi} \right) \right], \]  

\[ W_{ij} = \frac{1}{2} \left( U_{ij} - U_{i,j} \right), \quad \left( bS \right) = b_{ij} S_j^p = \frac{1}{2} \frac{P_k}{K}, \quad I_{ij} = -\frac{1}{2} b_{ij} b_{kl}, \quad C_s = 0.22; \]  

\[ C_1 = 3.4 + 1.8 \frac{P_k}{\varepsilon}, \quad d_1 = 0.2 - 0.325 \left( -2I_{ii} \right)^{1/2}, \quad d_2 = 0.3125, \quad d_3 = 0.1. \]  

The coefficients \( C_{e1}, C_{e2}, \) and \( C_\varepsilon \) are taken to be the same as in (5).

In [6, 7], the advantages and disadvantages of the model considered were analyzed. It was found that it permits prediction of a series of effects that cannot be described by turbulence models that use the notion of