PASSAGE OF AN ELECTROMAGNETIC PULSE THROUGH A LAYER OF HOMOGENEOUS PLASMA (EXACT SOLUTION)

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The authors found an exact analytical solution of the problems on reflected and transmitted waves in interaction of a short electromagnetic pulse with a plasma layer of finite extension. The problem is solved assuming the linearity of material equations. Analytical expressions of Green functions are obtained for the corresponding problems that allow one to write solutions for the transmitted and reflected waves with any shape of an incident wave. The exact solutions depend substantially on the plasma density and layer extension, which makes it possible to use them in processing experimental results for extraction of the above-mentioned parameters.

Introduction. Recent advances in laser technology have enabled the production of high-power ultrashort pulses, by means of which it is possible to virtually instantly ionize thin layers of a substance. These can be both foils of a solid substance and gas jets injected into vacuum, so that the range of densities of plasmas generated turns out to be rather extensive. The properties of such plasmas are investigated by means of sounding pulses of low power, whose propagation can be considered in a linear regime. In diagnosing, of great interest is the passage of short pulses (the plasma formed must have no time to fly apart) through the thin layers; moreover, the carrier frequency must be close to a plasma frequency. The latter implies that dispersion of the waves in the plasma should be strictly taken into account.

On the other hand, the comparatively small transverse dimension of the plasmas formed requires the correct consideration of boundary conditions on both edges of the plasma spacing. The problems of interaction of ultrashort pulses with thin layers of a substance were investigated in [1, 2]. The present work is devoted to a rigorous solution of the problem concerning the passage of a short laser pulse of arbitrary shape through thin plasma layers. A similar problem was solved in [3] but for a semi-infinite plasma, i.e., only one boundary condition was taken into account. Taking into account the second boundary not only complicates the solution, but also requires the elucidation of what kind of contribution the effects of rereflections make to the transmitted and reflected field and what the influence of the plasma-layer extension is.

We express the solution in terms of Green functions for the transmitted and reflected waves without restricting ourselves to a specific shape of the envelope of an incident pulse. In our approach, the expansion in terms of the so-called nonseparable solutions [4] of the wave equation naturally arises.

Statement of the Problem. An electromagnetic pulse is incident from infinity onto a layer of homogeneous isotropic plasma of finite extension. The pulse is normally incident onto the leading edge of the plasma (along the X axis); the electric field of the wave is linearly polarized. We consider a one-dimensional problem, i.e., there is no dependence of the dielectric permeability on the transverse coordinates Y and Z. The plasma is described in a linear approximation and has the dielectric permeability

\[
\varepsilon (\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \tag{1}
\]


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In vacuum, the fields $E_r(t, x)$ and $E_{\text{out}}(t, x)$ satisfy an ordinary homogeneous wave equation, while the field $E_t(t, x)$ in the plasma layer obeys the equation

$$\left( \partial_{xx}^2 - \frac{1}{c^2} \partial_t^2 - \frac{\omega_p^2}{c^2} \right) E_t(t, x) = 0,$$

which follows from Maxwell equations for a medium with dielectric permeability (1). Here we use the following notation: $\partial_{xx}^2$ and $\partial_t^2$ are the partial derivatives of the second order with respect to the spatial coordinate $x$ and time $t$, respectively; $\omega_p$ is the plasma frequency; $c$ is the velocity of light in vacuum.

The boundary conditions on the electric field follow from the continuity conditions for the electric and magnetic fields at both boundaries of the layer ($x = 0$ and $x = l$) (it is assumed that surface charges and currents are absent):

$$E_i(t, 0) + E_r(t, 0) = E_t(t, 0), \quad \partial_t E_i(t, 0) + \partial_x E_t(t, 0) = \partial_x E_i(t, 0),$$

$$E_i(t, l) = E_{\text{out}}(t, l), \quad \partial_t E_i(t, l) = \partial_x E_{\text{out}}(t, l).$$

At the initial instant of time, the pulse $E_i(t - x)$ is incident onto the leading edge of the plasma; the latter is in an unperturbed state, therefore, the initial conditions are as follows:

$$E_i(0, x) = E_i(0 - x) = E_i(-x), \quad E_t(0, x) = 0, \quad \partial_x E_i(0, x) = 0.$$

**Solution.** Let us pass to the dimensionless variables: $\omega_p t \rightarrow t$ and $\omega_p x \rightarrow x$. Then wave equation (2) takes the form

$$\left( \partial_{xx}^2 - \partial_t^2 - 1 \right) E_i(t, x) = 0.$$

To solve the last differential equation, we use the Laplace operator method. Using initial conditions (4) and taking into account the transformation properties that refer to the original time derivative, we obtain an equation equivalent to Eq. (5):

$$[\partial_{xx}^2 - (s^2 + 1)] \bar{E}_i(s, x) = 0.$$

We represent the solution of Eq. (6) in the form

$$\bar{E}_i(s, x) = \bar{E}_i^1(s, 0) \exp (-\sqrt{s^2 + 1} x) + \bar{E}_i^2(s, 0) \exp (\sqrt{s^2 + 1} x).$$

On the Laplace transform, boundary conditions (3) become as follows:

$$\bar{E}_i(s, 0) + \bar{E}_r(s, 0) = \bar{E}_t(s, 0), \quad \bar{E}_t(s, l) = \bar{E}_{\text{out}}(s, l),$$

$$\partial_x \bar{E}_i(s, 0) + \partial_x \bar{E}_r(s, 0) = \partial_x \bar{E}_t(s, 0), \quad \partial_x \bar{E}_t(s, l) = \partial_x \bar{E}_{\text{out}}(s, l).$$

Now we find the space-time dependences of the fields that propagate in vacuum. Since the incident pulse $E_i(t, x)$ moves from left to right (in the direction to the plasma), whereas the reflected pulse $E_r(t, x)$ moves from right to left (from the plasma), and finally the transmitted pulse $E_{\text{out}}(t, x)$ moves to the right, we have the dependences

$$E_i(t, x) = E_i(t - x), \quad E_r(t, x) = E_i(t + x), \quad E_{\text{out}}(t, x) = E_{\text{out}}(t - x),$$

which correspond to the following wave equations: