DERIVATION AND NUMERICAL SOLUTION
OF A CLOSED EQUATION FOR THE SPECIFIC
ISOSCALAR-SURFACE AREA IN A TURBULENT
REACTIVE FLOW

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Based on the equation obtained earlier for the joint probability density function of the fluctuations of an isotropic turbulent scalar field of a reagent and its gradient [Inzh.-Fiz. Zh., 71, No. 5, 827-849 (1998)] the authors derived and numerically solved an equation for the specific isoscalar-surface area $\Sigma(\Gamma)$ in a turbulent reactive flow. The equation for $\Sigma(\Gamma)$ contains the single-point probability density function for pulsations of a reactive scalar and the time function that depend on the distribution of the energy of turbulent velocity pulsations and the intensity of scalar reagent pulsations by different length scales. The corresponding equations are written for all these functions.

Introduction. The most significant approach to the modeling of combustion that allows for a deviation from the position of chemical equilibrium is based on the assumption which considers a turbulent flame as an ensemble of thin one-dimensional reaction zones (flamelets). Each zone is found in a locally laminar mixing environment. This description of the turbulent flame was named the laminar diffusion flamelet model. This concept was proposed for the first time in [2] and was extended in a number of papers (see, for example, [3]).

The instantaneous flame consists of localized reactive leafs that are transferred by a flow and are bent and stretched by turbulent vortices but remain identified structures.

The field of application of flamelet models has been the subject of discussion thus far. It is agreed that this concept is applicable in the region of large Damköhler numbers; in this case, the characteristic turbulence scales are much larger than the typical flame thickness. These conditions are satisfied in many practical situations, and the flamelet regime exists in spark-ignition engines, in continuous flows of aircraft engines, in rocket engines, and in industrial burners.

The most important class of flamelet models is associated with a balance equation for the density of the flame surface. This equation describes the transfer of the average reactive surface by a turbulent flow and also the mechanisms of production and destruction of the flame area. The notion of the flame area was already used in the previous combustion models. However, an equation for this quantity was proposed for the first time in [4], where the combustion of unmixed reactants at the early stage is controlled by the competition between the deformation of the elements of the flame and mutual annihilation of the flame area because of the destruction of its adjacent elements. The advantage of the flame model that is based on the equation for the surface area consists of using it to relate the analysis of an individual flamelet to that of the global turbulent field.

Having the calculated area of the flame surface $\Sigma_r$, we can easily calculate the average rate of heat release per unit volume or the flow rate of the reagent by the formulas

$$W = Q \Sigma_r, \quad W_i = -\frac{Q}{\Delta h_f} \Sigma_r.$$  

Here $Q$ is the rate of heat release per unit area of the flame; $\Delta h_f$ is the enthalpy.
In writing an equation for \( \Sigma_f \) based on intuitive arguments as was done in [4], we have to artificially introduce into the equation all the effects associated with the interaction of the rate of deformation and the combustion surface. At the same time, there is a possibility of deriving the equation for the flame surface on the basis of rigorous relations for the joint probability density function of a scalar and its gradient. Precisely this is the objective of the present investigation. In [5-7], the following formula that relates the specific isoscalar surface to the probability density function of a scalar and its gradient was proposed:

\[
\Sigma_{\omega_f}(\Gamma) = \int_0^\infty WP_{\omega_f}(W, \Gamma) dW.
\]  

The equation for \( P_{\omega_f}(\Gamma) \) is proposed in the work of Sosinovich et al. [1]. A solution for it remains to be found. However, it can be used as the reference point when an equation for the density function of the flame surface \( \Sigma_{\omega_f}(\Gamma) \) is derived.

A Closed Equation for the Joint Probability Density Function (JPDF) of the Fluctuations of a Reactive Scalar and Its Gradient. The equation for the JPDF \( P_t(W, \Gamma) \) was obtained in [1] for the case of an isotropic turbulent field (formula (117)). The function \( P_t(W, \Gamma) \) that describes the JPDF of the absolute value of the fluctuations of the scalar gradient and the fluctuations of the scalar \( F \) is related to \( P_t(W, \Gamma) \) as follows:

\[
P_t(W, \Gamma) = 4\pi W^2 P_t(W, \Gamma).
\]

We substitute the expression for \( P_t(W, \Gamma) \) from (3) into Eq. (117) from [1] and, having carried out simple manipulations, obtain the equation for \( P_t(W, \Gamma) \):

\[
\frac{\partial P_t(W, \Gamma)}{\partial t} = -DW^2 \frac{\partial^2 P_t(W, \Gamma)}{\partial \Gamma^2} +
\]

\[
+ \frac{S_U C(t)}{15 \nu} \left[ 1 + W \frac{\partial}{\partial W} \right] \left( 3 + W \frac{\partial}{\partial W} \right) P_t(W, \Gamma) -
\]

\[
- D N_t(\Gamma) \left[ \frac{2}{W^2} - \frac{2}{W} \frac{\partial}{\partial W} + \frac{\partial^2}{\partial W^2} \right] P_t(W, \Gamma) -
\]

\[
- 2D \frac{\partial}{\partial \Gamma} \left( X_t(\Gamma) \left[ 1 + W \frac{\partial}{\partial W} \right] P_t(W, \Gamma) \right) -
\]

\[
- \left[ \omega (\Gamma) \frac{\partial}{\partial \Gamma} + \frac{\partial \omega (\Gamma)}{\partial \Gamma} \left( 2 + W \frac{\partial}{\partial W} \right) \right] P_t(W, \Gamma)
\]

with the initial

\[
P_t(W, \Gamma) \big|_{t=0} = P_0(W, \Gamma)
\]

and boundary conditions

\[
P_t(W, \Gamma) \big|_{W=\infty} = 0, \quad P_t(W, \Gamma) \big|_{W=0} = 0, \quad \frac{\partial P_t(W, \Gamma)}{\partial \Gamma} \big|_{\Gamma=0} = 0, \quad P_t(W, \Gamma) \big|_{\Gamma=\Gamma_{\text{min}}} = 0.
\]