The stress intensity factors (SIFs) are evaluated for flat elliptical cracks located in a transversally isotropic material (cracks are assumed perpendicular to the transtropy axis) under an arbitrary load and symmetric temperature. The SIFs for an elliptical crack in a transversally isotropic medium are determined using the formulas (derived by the author in his previous studies) of transition from an isotropic to transversally isotropic material and the relative problem for an isotropic medium. It is proved that these formulas can be employed for an arbitrary homogeneous transversally isotropic material (no matter whether the roots of some characteristic equation of the material are real or complex) with an arbitrary flat crack or a system of coplanar flat cracks, including elliptical ones, under an arbitrary load and symmetric temperature. A transversally isotropic material with two coplanar elliptical cracks is considered as an illustrative example. The dependences of the SIFs on the parameters of cracks and their arrangement at a decreasing temperature are presented.

The stress intensity factors in elastic isotropic and transversally isotropic media containing elliptic cracks under loading were analyzed in [1-3, 5, 9, 12, 15-22, etc.]. Different three-dimensional problems on a temperature action in a transversally isotropic medium with an elliptic crack were solved in [6, 10-14]. The stress intensity factors in a material with multiply connected flat cracks were analyzed in [7, 8]. For a crack located in the isotropy plane of a transversally isotropic medium under loading, a rather simple relationship between similar problems for a flat crack in isotropic and transversally isotropic media is presented in [17]. The connection between the stress intensity factors in problems of thermoelasticity for isotropic and transversally isotropic media under a symmetric temperature action was established in [6]. In the present study, the applicability of the results from [6] to an arbitrary homogeneous transversally isotropic material (no matter whether the roots of some characteristic equation are real or complex) is proved, and they are used to evaluate the stress intensity factors of both isotropic and transversally isotropic media with an elliptic crack under arbitrary loading and a symmetric temperature action. The results of [18, 22] are used as the solution of the basic purely elastic problem for an isotropic medium.

Note that the determination of the stress intensity factors for elliptic cracks is of special interest, since they, first, allow us to solve the problem in a closed form, and, second, the stress intensity factors for circular, tunnel, parabolic, and other crack geometries follow from the expressions for elliptic forms as special cases. This shape of the crack also allows us, fitting properly the curvature of the ellipse, to effectively approximate a smooth crack of more complicated geometry at any point.

First, let us briefly present the results of [22]. Let a crack occupy a domain $\Omega_1$ inside the ellipse

$$x_1^2/a_1^2 + x_2^2/a_2^2 = 1, \quad a_1^2 > a_2^2,$$

in the plane $x_3 = 0$. Introducing ellipsoidal coordinates $\xi_\alpha (\alpha = 1, 2, 3)$, which are the roots of the cubic equation $\omega (\xi_\tau) = 0$,
we can determine the boundary of the ellipse in the plane \( x_3 = 0 \) by assuming that \( \xi_3 = 0 \), \( \xi_2 = 0 \) and the crack surface by assuming that \( \xi_3 = 0 \). The stress state is determined using the Trefftz representations in terms of three harmonic functions \( f_\alpha \) (\( \alpha = 1, 2, 3 \)), for which we have

\[
\sigma_{33} = -2 \mu f_{3, 33}, \\
\sigma_{\alpha 3} = -2 \mu \left[ (1 - \nu) f_{\alpha, 33} - \nu (f_{1,1} + f_{2,2} + \alpha) \right] \quad (\alpha = 1, 2).
\]

It is essential in constructing \( f_\alpha \) (\( \alpha = 1, 2, 3 \)) to use the system of harmonic functions

\[
V_n = \int_{\xi_3} \left[ \omega (s) \right]^n \frac{ds}{\sqrt{Q(s)}} \quad n = 1, 2, ....
\]

\[
Q(s) = (a_1^2 + s) (a_2^2 + s) (a_3^2 + s).
\]

When a nonpolynomial-type load is applied to the crack surface, it is proposed to approximate it by polynomial loads (for which the exact solution is known) by "zeroing" the error of approximation on the crack surface using the least-squares method.

In the case of symmetric polynomial loading, the boundary conditions have the form

\[
\sigma_{13} (x_1, x_2, x_3) = \sigma_{23} (x_1, x_2, x_3) = 0, \quad |x_1| < \infty, \quad |x_2| < \infty,
\]

\[
\sigma_{33} (x_1, x_2, 0) = p (x_1, x_2), \quad (x_1, x_2) \in \Omega,
\]

\[
u_{3} (x_1, x_2, 0) = 0, \quad (x_1, x_2) \in \Omega_1,
\]

where

\[
p (x_1, x_2) = \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{m=0}^{M} \sum_{n=0}^{m} A_{i,j,m-n,n} x_1^{2m-2n+1} x_2^{2n+j}.
\]

According to \([19]\), the harmonic function \( f_3 \) can be represented in the form

\[
f_3 = \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{M} \sum_{l=0}^{k} C_{i,j,k-l,l} F_{2k-2l+i,2l+j}
\]

where

\[
F_{mn} = \int_{\xi_3} \partial_1^m \partial_2^n \omega^{m+n+1} \frac{ds}{\sqrt{Q(s)}}^{1/2},
\]

\[
Q(s) = (a_1^2 + s) (a_2^2 + s) (a_3^2 + s).
\]