The study is made of the delayed fracture of a viscoelastic orthotropic plate caused by subcritical advancement of a rectilinear microcrack, which is located along one of the orthotropic axes. The crack develops because of stretching of the plate by uniformly distributed increasing and cyclic external forces perpendicular to the crack line. The investigation is carried out within the framework of the Boltzmann–Volterra theory for resolvent integral operators of difference type, which describe the deformation of a material with time-dependent rheological properties. The analytical form of the kernel of an irrational function of a linear combination of the above integral operators is determined by the method of operator continued fractions. Numerical calculations are conducted for resolvent bounded integral operators with a kernel in the form of Rabotnov’s fractional–exponential function. The kinetics of growth of a crack with tip regions commensurable with the crack length is studied. A comparison with the results obtained within the framework of the concept of the thin structure of the crack tip is given.

In the present paper, the delayed fracture of a viscoelastic orthotropic plate due to subcritical development of a rectilinear through crack with a considerable prefracture zone (microcrack) located along one of the orthotropy axes is examined based on the modified δC-model of fracture from [3] and within the framework of the concept on constancy of the size of the prefracture zone near the front of an advancing crack. The development of the crack due to the long-term action on the plate of uniformly distributed monotonically increasing and cyclic external loads normal to the crack plane is described. Studies are carried out within the framework of the Boltzmann–Volterra theory for resolvent integral operators of difference type, which describe the deformation of a material with time-dependent rheological properties. The operator continued-fraction method developed in [3, 4, 6] is used to determine the analytical form of the kernel of an irrational function of a linear combination of the above integral operators. The numerical calculations are carried out for resolvent bounded operators with a difference kernel as Rabotnov’s fractional–exponential function [9]. The kinetics of growth of cracks with tip regions commensurable with the crack length (microcracks), when the introduction of stress intensity factors becomes meaningless, is investigated. A comparison is made with calculated results for cracks with small prefracture zones (macrocracks) when the concept of the thin structure of the crack tip is valid and it is possible to introduce stress intensity factors.

1. Formulation of the Problem. Let us consider a thin plate made of an orthotropic viscoelastic material. We examine the delayed fracture of such a plate weakened by a rectilinear through crack of normal fracture located along one of the orthotropy axes (along the x-axis) and subject to the action of tensile forces, which are uniformly distributed, normal to the crack plane, and have intensity

\[ p(t) = p_0 P(t), \]  

where

(a) \( P(t) = 1 + at^\chi, \ a > 0, \ \chi > 0 \) (a monotonically increasing load)

(b) \( P(t) = 1 + \gamma \sin \omega t; \ 0 < \gamma \ll 1 \) (a cyclic load).
In the case of a cyclic load, we assume that the static part of the load \( p_0 \) is much greater than its cyclic component (the action of tensile forces on the plate is being considered); therefore, the change in the crack length due to creep of the material will be nondecreasing.

Then, according to the Volterra principle, the equation of the crack contour in a viscoelastic material can be presented as [3, 8]

\[ \delta(x, t) = L^* \delta_0(\sigma(t), x, l(t)) , \]  

(1.2)

where

\[ \delta(\sigma(t), x, l(t)) = L_0 \delta_0(\sigma(t), x, l(t)) = 2L_0 \sigma(t) \Phi(x, l(t)) \]  

(1.3)

is the elastic opening of a crack of length \( l(t) \) at a point \( x \) \( l(t) \leq x \leq l(t) + d \) and \( d \) is the length of the prefraction zone,

\[ \Phi(x, l(t)) = (x - l(t)) \Gamma(x, l(t)) - (x + l(t)) \Gamma(x, -l(t)) , \]  

(1.4)

\[ \Gamma(x, l(t)) = \ln \frac{\left( \frac{l(t)}{l(t) + d} \right)^2 - x l(t) - \varphi(x, l(t))}{\left( \frac{l(t)}{l(t) + d} \right)^2 - x l(t) + \varphi(x, l(t))} , \]  

(1.5)

\[ \varphi(x, l(t)) = \sqrt{\left( l(t)^2 - x^2(t) \right) \left( l(t) + d \right)^2 - l^2(t) .} \]  

(1.6)

The operator \( L^* \) is represented as [3, 4]

\[ L^* = \frac{1}{\pi \sqrt{E_{11}^* E_{22}^*}} \sqrt{2 \left( \sqrt{\frac{E_{11}^*}{E_{22}^*}} - \frac{\nu_{12}^*}{E_{11}^*} \right) + \frac{E_{11}^*}{G_{12}^*}} . \]  

(1.7)

If the viscoelastic properties of the material of the body can be described by the resolvent operators

\[ \frac{1}{E_{11}^*} = \frac{1}{E_{11}^0} \left( 1 + \lambda_1 R^* (\beta_1) \right) , \]  

(1.8)

\[ \frac{1}{E_{22}^*} = \frac{1}{E_{22}^0} \left( 1 + \lambda_2 R^* (\beta_2) \right) , \]

\[ \frac{1}{G_{12}^*} = \frac{1}{G_{12}^0} \left( 1 + \lambda_G R^* (\beta_G) \right) , \]

\( \nu_{21} = \nu_{21}^0 \left( 1 + \lambda_\nu R^* (\beta_\nu) \right) \)

(1.9)

where

\[ \lambda_1, \lambda_2, \lambda_G, \beta_1, \beta_2, \beta_G, \text{ and } \beta_\nu \text{ are determined from creep tests}, \]

then the irrational function (1.7) can be determined by using the operator continued-fractions method [3, 6], according to which (1.7) can be approximated by the expression

\[ L^* = L^0 \left( 1 + \sum_{i=1}^{L} k_i R^* (\lambda_i) \right) . \]  

(1.9)

Then

\[ R^* (\lambda) \cdot 1 = \int_{\lambda}^{t} R(t - \tau, \lambda) d \tau . \]  

(1.10)