THE STABILITY OF THE INTERFACE BETWEEN TWO BODIES COMPRESSED ALONG INTERFACE CRACKS. 2. EXACT SOLUTIONS FOR THE CASE OF EQUAL ROOTS

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The problem of buckling of the interface between two bodies is considered for the case where several plane cracks are located in the interface and the bodies are compressed along the cracks (along the interface of two different materials). The studies were carried out for a plane problem using the three-dimensional linearized theory of stability of deformable bodies. The complex variables and potentials of the above-mentioned linearized theory are used. This problem is reduced to the problem of linear conjugation of two analytical functions of a complex variable. The exact solution of the above-mentioned buckling problem is obtained for the case where the roots of the basic equation are equal. Some mechanical effects are analyzed under general conditions (elastic, elastoplastic, compressible, incompressible, isotropic, and orthotropic bodies).

In [6], plane problems on the stability of the interface between two bodies compressed along cracks located in the interface were formulated, and the exact solution of the basic equation with unequal roots was given. The exact solution of the basic equation with equal roots was derived in [7]. In the mentioned papers, the method of reduction to a linear-conjugation problem for two functions of complex variables was applied, and the stresses and displacements for a half-plane were represented (within the framework of a plane problem of the three-dimensional linear theory of stability of deformable bodies [3]) in terms of one function of complex variables, which is holomorphic in the entire plane. In deriving the exact solutions, complex stresses and displacements were used when the boundary conditions are satisfied on the sections of complete adhesion. In this connection, it was pointed out in [13] (a remark on page 31) that, in essence, these exact solutions pertain to the case where the materials of the lower and upper half-planes are the same, i.e., pertain to the case [1, 2, 4, 5]. In [19], additional information concerning the formulation of problems is given, and the exact solution for the interface between two different materials is obtained when the roots of the basic equation are unequal ($\mu_1 \neq \mu_2$). In [19], the method of reduction to the linear-conjugation problem for two functions of complex variables was used, and the stresses and displacements for a half-plane were expressed in terms of one function holomorphic in the entire plane, i.e., the results obtained in [6] were employed. In the present paper, taking into account the formulation of problems in [6, 19] and the results of [7], we derive the exact solution for the interface of two different materials for the case of equal ($\mu_1 = \mu_2$) roots of the basic equation.

1. Basic Relations. A calculation procedure that completely coincides with that from [6, 7, 19] is being considered. By the statement of problems (for example, according to (1.3) [6]) on the interface for $y_2 = 0$, we obtain the following homogeneous mixed boundary conditions:

$$Q_{22}^{(\pm)} = 0, \quad Q_{21}^{(\pm)} = 0 \quad \text{for} \quad y_1 \in L_1 \quad \text{and} \quad y_2 = 0,$$

$$Q_{22}^{(+)} = Q_{22}^{(-)}, \quad Q_{21}^{(+)} = Q_{21}^{(-)}, \quad \frac{\partial}{\partial y_1} (u_1^{(+)}) - u_1^{(-)}) = 0.$$
\[
\frac{\partial}{\partial y_1} (u_2^{(+)} - u_2^{(-)}) = 0 \quad \text{for} \quad y_1 \in L_2 \quad \text{and} \quad y_2 = 0.
\] (1.1)

Hereafter, we consider that the roots of the basic equation are equal [2, 4, 5]:
\[
\mu_1 = \mu_2, \quad \text{Re} \mu_1 = 0, \quad \mu_1 = -\mu_1, \quad z_1 = y_1 + \mu_1 y_2.
\] (1.2)

In [7], the complex stresses and displacements for a half-plane were expressed (within the framework of a plane problem of the three-dimensional linear theory of stability of deformable bodies [3]) in terms of one complex-variable function \( \Phi(z_1) \), which is holomorphic in the entire plane, the roots (1.2) of the basic equation being equal. Following (2.5) from [7], we can present these results as follows:

\[
Q_{22} + (\mu_1 \gamma_{21}^{(1)})^{-1} Q_{11} = D_{11} \left[ \Phi(z_1) + \Phi(\bar{z}_1) \right],
\]

\[
Q_{22} - (\mu_1 \gamma_{21}^{(1)})^{-1} Q_{21} = D_{21} \left[ \Phi(z_1) - \Phi(\bar{z}_1) \right] + (z_1 - \bar{z}_1) \Phi'(z_1).
\]

\[
2 \frac{\partial}{\partial y_1} \left( u_1 - \frac{\gamma_1^{(1)}}{\gamma_2^{(1)}} u_2 \right) = D_{31} \Phi(z_1) + D_{32} \Phi(\bar{z}_1) + D_{33} \Phi'(z_1) + 2 \gamma_1^{(1)} (z_1 - \bar{z}_1) \Phi'(z_1).
\] (1.3)

The quantities
\[
D_{mn} = D_{nm} (\gamma_p^{(q)}, \mu_1, \gamma_{pq}^{(k)})
\] (1.4)

are explicitly determined by relations (2.6) from [7]. The quantities \( \mu_1, \gamma_m^{(k)} \), and \( \gamma_{nm}^{(k)} \) are determined by the relations from [2, 4, 5] in terms of the components of the tensor \( \omega \) for compressed bodies and in terms of the components of the tensor \( \kappa \) for incompressible bodies. Thus, expressions (1.3) determine complex stresses and the displacements in the upper or lower half-planes \( D^{(\pm)} \) (all quantities should be marked with the superscript \( \pm \)) through one function \( \Phi^{(+)}(z_1) \) for the upper half-plane and \( \Phi^{(-)}(z_1) \) for the lower half-plane, which is holomorphic in the entire plane. It should be noted that as \( \sigma_1^{0(\pm)} \to 0 \), representation (1.3) turns into the well-known representation of the classical linear theory of elasticity of an isotropic body obtained in [12]. Note also that for the case of equal roots (1.2), the following conditions are satisfied in accordance with (2.270) in [5] for compressible and incompressible bodies:

\[
\text{Im} \gamma_{22}^{(2)} = 0, \quad \text{Im} \gamma_{11}^{(1)} = 0, \quad \text{Im} \gamma_{11}^{(2)} = 0, \quad \text{Im} \gamma_{21}^{(1)} = 0,
\]

\[
\text{Re} \gamma_{21}^{(2)} = 0, \quad \text{Re} \gamma_{12}^{(2)} = 0, \quad \text{Im} \gamma_{21}^{(1)} = 0, \quad \text{Im} \gamma_{1}^{(2)} = 0.
\]

\[
\text{Re} \gamma_{2}^{(1)} = 0, \quad \text{Re} \gamma_{2}^{(2)} = 0, \quad \gamma_{11}^{(1)} = -\gamma_{21}^{(1)}.
\] (1.5)

It is important to note that in (2.6) [7], a misprint slipped into the expression for the quantity \( D_{32} \), which is determined as follows:

\[
D_{32} = \gamma_1^{(2)} + 2 \gamma_1^{(1)} + \frac{\gamma_2^{(1)} \gamma_2^{(2)}}{\gamma_2^{(1)}} - \gamma_2^{(1)} \mu_1 \gamma_2^{(1)} \gamma_2^{(2)} + \gamma_2^{(2)} \frac{\gamma_2^{(1)}}{2 \mu_1 \gamma_2^{(1)}}.
\] (1.6)

From (1.4), the quantities that enter the boundary conditions (1.1) can be determined in the form

\[
Q_{22} = C_{11} \left[ \Phi(z_1) - \Phi(\bar{z}_1) \right] + \bar{C}_{11} \left[ \Phi(z_1) - \Phi(\bar{z}_1) \right]
\]

\[
+ C_{12} (z_1 - \bar{z}_1) \left[ \Phi'(z_1) - \Phi'(\bar{z}_1) \right].
\]