ON THE THERMOMECHANICAL BEHAVIOR OF A ROUND FLEXIBLE ORTHOTROPIC PLATE

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The critical temperature of an orthotropic round plate at which it loses stability is determined, and its deflection is studied on the basis of flexible plate thermoelasticity. The plate is under conditions of heat exchange with the environment. The approximate method is based on the representation of stresses and displacements in the form of algebraic polynomials.

The loss of stability of the flat shape of a nonuniformly heated round orthotropic plate is studied in [2]. The plate buckles when the temperature on the inner contour reaches a critical value. However, the deflection of the plate caused by this deformation process is not examined in [2], even though this deflection can be substantial, especially in the case of flexible plates.

Here, using the equations of thermoelasticity of flexible plates, we study the large deflections of thin plates nonuniformly heated along the radius when the critical temperature is reached. The approximate method is based on the representation of stresses and displacements in the form of algebraic polynomials.

1. The plate temperature is determined by solution of the equation of stationary thermal conductivity

\[
\frac{d^2 T}{d \rho^2} + \frac{1}{\rho} \frac{d T}{d \rho} - \beta^2 (T - T_2) = 0
\]

under the conditions

\[
T = T_0 \quad (\rho = \rho_0) \quad T = T_1 \quad (\rho = 1)
\]

and has the form

\[
T = T_2 + \frac{1}{u_0 (\beta \rho_0)} \left\{ (T_0 - T_2) u_0 (\beta \rho) - (T_1 - T_2) u_1 (\beta \rho) \right\}, \quad (1.1)
\]

where

\[
u_0 (\beta \rho) = K_0 (\beta \rho) I_0 (\beta \rho) - I_0 (\beta) K_0 (\beta \rho),
\]

\[
u_1 (\beta \rho) = K_0 (\beta \rho_0) I_0 (\beta \rho) - I_0 (\beta \rho_0) K_0 (\beta \rho),
\]

\[
\beta = \frac{2 \alpha R^2}{h \kappa_r}, \quad \rho = \frac{r}{R}, \quad \rho_0 = \frac{r_0}{R}.
\]

In the absence of heat exchange on the lateral surfaces of the plate (\(\alpha = 0\)), from expression (1.1) follows the relation

\[
T = T_2 + \frac{1}{u_0 (\beta \rho_0)} \left\{ (T_0 - T_2) u_0 (\beta \rho) \right\},
\]

To obtain the equations that determine the stress–strain state of a flexible orthotropic plate, we use the relations

\[ \varepsilon_r = \frac{du}{dr} + \frac{1}{2} \theta'^2 + \frac{z}{r} \frac{d\theta}{dr} = \frac{1}{E_1} \sigma_r - \frac{v_2}{E_2} \sigma_\varphi + \alpha_r (T - T_c), \]

\[ \varepsilon_\varphi = \frac{u}{r} + \frac{z}{r} \theta = \frac{1}{E_2} \sigma_\varphi - \frac{v_1}{E_1} \sigma_r + \alpha_\varphi (T - T_c), \]

from which we find

\[ \frac{du}{dr} + \frac{1}{2} \theta'^2 = \frac{1}{h} \left( \frac{1}{E_1} N_r - \frac{v_2}{E_2} N_\varphi \right) + \alpha_r (T - T_c), \]

\[ \frac{u}{r} = \frac{1}{h} \left( \frac{1}{E_2} N_\varphi - \frac{v_1}{E_1} N_r \right) + \alpha_\varphi (T - T_c) \quad (2.1) \]

and

\[ \frac{d\theta}{dr} = \frac{12}{h^3} \left( \frac{1}{E_1} M_r - \frac{v_2}{E_2} M_\varphi \right), \]

\[ \frac{\theta}{r} = \frac{12}{h^3} \left( \frac{1}{E_2} M_\varphi - \frac{v_1}{E_1} M_r \right) \quad (2.2) \]

where the angle of inclination \( \theta = -\frac{d\vartheta}{dr} \).

The equation of deformation consistency is

\[ \varepsilon_r - \frac{d}{dr} (r \varepsilon_\varphi) = \frac{1}{2} \theta'^2. \quad (2.3) \]

It follows from (2.1) and (2.2) that

\[ N_r = D_N \left[ \frac{du}{dr} + \frac{1}{2} \theta'^2 + v_2 \frac{u}{r} - (\alpha_r + v_2 \alpha_\varphi) (T - T_c) \right], \]

\[ N_\varphi = D_N E \delta_E \left[ v_1 \left( \frac{du}{dr} + \frac{1}{2} \theta'^2 \right) + \frac{u}{r} - (v_1 \alpha_r + \alpha_\varphi) (T - T_c) \right] \quad (2.4) \]

and

\[ M_r = D_M \left( \frac{d\theta}{dr} + v_2 \frac{\theta}{r} \right), \quad M_\varphi = D_M E \delta_E \left( \frac{\theta}{r} + v_1 \frac{d\theta}{dr} \right). \quad (2.5) \]

Here,

\[ D_N = \frac{h E_1}{1 - v_1 v_2}, \quad D_M = \frac{h^3 E_1}{12 (1 - v_1 v_2)}, \quad \delta_E = \frac{E_2}{E_1}. \]

The equilibrium equations are