THE ELECTROELASTICITY PROBLEM FOR PIEZOCERAMIC MEDIA
WITH BILATERAL HYPERBOLIC TUNNEL CAVITIES

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An explicit solution of the static problem of electroelasticity is obtained for a transversally isotropic medium that contains bilateral hyperbolic tunnel cavities. It is assumed that the plane of isotropy of the medium coincides with the plane of symmetry of the medium, and also that the surfaces of the cavities are free of mechanical forces and that the normal component of the electric induction vector is equal to zero on the cavities. A uniform tensile force and difference in electric potentials are specified at a sufficient distance from the cavities in a direction perpendicular to the plane of isotropy of the medium.

A solution of the corresponding problem for a piezoceramic medium containing external bilateral rectilinear cracks is obtained as a special case.

Numerous published studies have been concerned with the investigation of static and dynamic problems of electroelasticity [1-4, 6-9]. However, the above problem has not been considered in any of the known published studies.

Statement of Problem. We direct the z-axis along the axis of anisotropy of the medium and situate the x- and y-axes in the plane of isotropy. Let us assume that the two axes of the hyperbola coincide with the x- and z-axes.

Let us suppose that the components of the displacement vector of the medium \( \mathbf{u} = (u, 0, w) \) and the electric potential vector \( \mathbf{\psi} \) are functions of \( x \) and \( z \). Then the complete system of equations of statics of piezoceramic media will contain the following equations [2]:

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0; \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0; \quad (1)
\]

equations of induced electrostatics

\[
\text{div} \, \mathbf{D} = 0; \quad \mathbf{E} = -\mathbf{\nabla} \mathbf{\psi}; \quad (2)
\]

equations of state

\[
\sigma_x = e_{11}^E u_x + e_{13}^E u_z - e_{31}^E E_z; \quad \sigma_z = e_{13}^E u_x + e_{33}^E u_z - e_{33}^E E_z;
\]

\[
\tau_{zx} = \tau_{xz} = e_{44}^E \varepsilon_{xz} - e_{15}^E E_x; \quad D_x = e_{11}^s E_x + e_{15}^s \varepsilon_{xz};
\]
Cauchy relations

\[ \varepsilon_x = \frac{\partial u}{\partial x} ; \quad \varepsilon_z = \frac{\partial w}{\partial z} ; \quad \varepsilon_{wx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} . \]  

Substitution of the expressions in (3) and (4) into Eqs. (1) and (2) leads to a system of three equations of equilibrium written for the displacements \( u \) and \( w \) and the electric potential \( \psi \). Its solution may be represented in the following form [5]:

\[ u = \sum_{j=1,2,3} \frac{\partial \Phi_j}{\partial x} ; \quad w = \sum_{j=1,2,3} k_j \frac{\partial \Phi_j}{\partial z} ; \quad \psi = \sum_{j=1,2,3} l_j \frac{\partial \Phi_j}{\partial z} . \]  

The functions \( \Phi_j \) must satisfy the following equations:

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \Phi_j = 0 ; \quad z = \sqrt{v_j} z_j \quad (j = 1, 2, 3) . \]

where the quantities \( v_1 \), \( v_2 \), and \( v_3 \) are the roots of the equation

\[ v^3 (A_2 + B_2 - C_1 D_2) + v^2 (A_1 B_3 + A_2 B_2 - C_1 D_3 - C_2 D_2) + v (A_2 B_3 + A_3 B_2 - C_2 D_3 - C_3 D_2) + A_3 B_3 - C_3 D_3 = 0 . \]  

Here, the following notation has been introduced:

\[ A_1 = e_{11}^E e_{15}^E ; \quad A_2 = (c_{44}^E + c_{13}^E) (e_{31}^E + e_{15}^E) - c_{11}^E e_{33}^E - c_{44}^E e_{15}^E ; \]
\[ A_3 = c_{44}^E e_{33}^E ; \quad B_2 = - \left[ v_{11}^S \left( c_{13}^E + c_{44}^E \right) + e_{33}^E (e_{31}^E + e_{15}^E) \right] ; \]
\[ B_3 = e_{33}^S (c_{13}^E + c_{44}^E) + e_{33}^E (e_{31}^E + e_{15}^E) ; \quad C_1 = - c_{11}^E e_{15}^S ; \]
\[ B_3 = (e_{15}^E + e_{31}^E)^2 + c_{11}^E v_{33}^S + c_{44}^E e_{15}^S ; \quad C_3 = - c_{44}^E e_{33}^S ; \]