SUBCRITICAL GROWTH OF HIGH-CYCLE FATIGUE CRACKS
IN FINITE THIN ISOTROPIC PLATES*

V. P. Golub and E. A. Panteleev

The problem of constructing a two-stage model of the motion of a fatigue crack in finite thin isotropic plates under symmetric tension-compression is formulated, and a method for its solution is considered. The two-stage nature is regarded as the presence of the incubation and propagation stages. The model is constructed by jointly considering the resolving equations of the theory of elasticity and the evolutionary equations of the mechanics of continuous damage. The damage function's attaining the critical value is considered as a criterion of initiation of local fracture and movement of a fatigue crack. Plates containing central and lateral cracks are considered. The effect of the level of stresses, the finiteness of the plate, the initial length of the crack, and the behavior of the length of the plastic zone on fracture kinetics is evaluated.

The problem of fatigue fracture of flat structural elements containing initial sharp-tipped imperfections, such as cracks, under monoaxial symmetric high-cycle loading is considered. Fatigue fracture is regarded as a two-stage process involving the incubation and advancement of a fatigue crack. The model of incipient growth and propagation of a crack is constructed by solving a boundary-value problem of the theory of elasticity with a moving boundary. The law of motion of the boundary is specified by the kinetics of accumulation of dispersed fatigue damages. To analyze the stress state at the tip of an advancing crack, a modified boundary-element method is used. Problems of growth of fatigue cracks in finite thin isotropic plates with stress concentrators in the form of lateral and internal cracks are solved.

1. Introduction. The problem of fatigue fractures pertains to the most urgent and complex problems of modern fracture mechanics and is of theoretical and practical significance. Extensive experimental data were accumulated in this field, and, on this basis, the laws of fatigue fracture were established, and useful empirical relations were formulated [1, 8, 13–15].

An important stage in the study of the fatigue phenomenon was understanding the fact that fatigue fracture, at least, represents a two-stage process consisting of the stage of hidden fracture and the stage of propagation of a fatigue crack. These new tendencies stimulated the development of new approaches to the study of the fatigue process, and the traditional experimental fatigue curves were supplemented by experimental diagrams of growth of fatigue cracks. As a result, the strength of materials and structural elements began to be analyzed from their resistance to the propagation of cracks on the basis of the crack-resistance characteristics [10, 14].

One of the major advances in this field was, apparently, the possibility of solving problems of high-cycle fatigue within the framework of the classical formulation of the mechanics of a deformable solid [1, 2, 7, 16]. The conditions for formation and motion of fatigue cracks were analytically correlated with the analysis of the stress–strain state in the vicinity of a crack tip, and, as a result, a complete system of equations specifying the kinetics of fatigue fracture was constructed. The condition of reaching by the stress intensity factor the critical value at the crack front according to the Paris–Erdogan law

* The present work was carried out with financial support from the European Community in accordance with the INTAS-UA 95-0202 International Project.

---

Another approach to the simulation of the nucleation and growth of high-cycle fatigue cracks is proposed in [3–5, 18, 19]. The model of fatigue fracture is constructed in this case also based on joint consideration of the resolving equations of the theory of elasticity and the equation describing the motion of the fracture front. However, to allow for time effects, models of fracture involve the kinetic equations for the damage function rather than the kinetic equations for the stress intensity factor. The fundamental possibility of solving dynamic problems of crack mechanics on the basis of the mechanics of continuous damage was formulated by L. M. Kachanov [7].

The authors of [3–5, 18, 19] constructed models of growth of fatigue cracks in infinite plates for which analytical expressions for the components of the stress tensor in the vicinity of a crack tip are known. In the present study, this approach is extended to finite cracked plates with unknown stress distribution.

2. Two-Stage Model of Fatigue Fracture. Fatigue fracture is modeled here in the context of the phenomenological approach, which assumes describing the phenomenon being examined by using the macroscopic characteristics and relationships between them, proved by a macroexperiment.

2.1. Initial Relations. We are considering a thin plate (Fig. 1) of width 2a and length 2b made of an isotropic linearly elastic material subjected to long-term loading by cyclically varying tension–compression stresses

\[ \bar{p} = \sigma_a \sin (2 \pi n) \Rightarrow \bar{p} = \sigma_a \sin (2 \pi f t), \]

which are uniformly distributed along the plate side of length 2a. Here, \( \sigma_a \) is the nominal amplitude of cyclic stress, \( n \) and \( f \) are the number of cycles and the frequency of variation in \( \sigma_a \), respectively, and \( t \) is time. High-cycle loading is usually realized for values of \( \sigma_a \) less than the yield point of the material \( \sigma_y \) and for \( f > 10 \) Hz, and the number of cycles to fracture is of order \( 10^5 \)–\( 10^8 \). The plate contains an initial central or edge crack of normal separation with initial \( l_0 \) and current \( l_f \) half-lengths (for the central crack) or lengths (for the edge crack). In accordance with the Leonov–Panasyuk–Dugdale model [9, 17], a crack is considered as a narrow slit with radius of curvature \( \rho \rightarrow 0 \) at the front and with an indefinitely thin plastic tip region of length \( \lambda \) ahead of the front. The length of the tip region \( \lambda \) for high-cycle fatigue cracks can be determined by the Rice relation [21]

\[ \lambda = \left( \frac{\sigma_{\text{max}}}{\sigma_y} \right)^2 \cdot \frac{l}{3} \Rightarrow \lambda = \left( \frac{\sigma_a}{\sigma_y} \right)^2 \cdot \frac{l}{3}, \]

where \( \sigma_{\text{max}} \) is the maximum stress in the vicinity of the crack tip, \( \sigma_y \) is the yield stress, \( l \) is the crack length, and \( \lambda \) is the length of the plastic tip region.