INTERACTION OF HARMONIC ELASTIC WAVES WITH CLOSELY SITUATED ARBITRARILY ORIENTED CRACKS

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UDC 539.375

The interaction of plane harmonic elastic waves with closely situated arbitrarily oriented cracks is considered. The contact pressure forces of the edges and friction in their contact area are taken into account. An algorithm for numerical iteration solution is constructed and some effects of the mutual influence of cracks are investigated.

It is well known that for static loading, the mutual effect of cracks decreases as the distance between them increases [8, 10], and for dynamic loading, this dependence acquires an undulating character [3—6]. The mutual effect of cracks was studied earlier for collinear cracks of the same length on which a harmonic compression wave was orthogonally incident [3, 4, 6]. Effects associated with the orientation of the cracks with respect to the direction of wave propagation and relative to one another remain unstudied. An algorithm for mathematical modeling of the interaction of plane elastic waves with closely situated cracks was developed on the basis of the ideas of [5].

We consider an elastic plane \( R^2 = \{ x : x_3 = 0 \} \) that has been weakened by two rectilinear cracks \( \Omega_1 \) and \( \Omega_2 \) with lengths \( 2L_1 \) and \( 2L_2 \), respectively,

\[
\begin{align*}
\Omega_1 &= \{ x : \| x \| \leq L_1 , \ x_2 = 0 \}, \\
\Omega_2 &= \{ x : x_1 = h_1 + \cos ( \alpha ) \cdot t , \ x_2 = h_2 + \sin ( \alpha ) \cdot t , \ \| t \| \leq L_2 \} .
\end{align*}
\]

(1)

Propagating in the plane are harmonic compression and shear waves whose potential function

\[
\varphi_\alpha ( x , t ) = \varphi_\alpha^0 e^{i ( k_\alpha ( n , x ) - \omega t )} , \quad \alpha = 1 , 2 ,
\]

(2)

where \( \varphi_\alpha^0 \) is the wave amplitude, \( n = ( \cos \gamma , \sin \gamma ) \) is the vector of the direction of wave propagation, and \( \omega \) is the cyclic frequency. For the compression wave, \( k_1 = \frac{\omega}{c_1} \), where \( c_1 = \frac{\sqrt{\lambda + 2 \mu}}{\rho} \) is the wave velocity; for the shear wave, \( k_2 = \frac{\omega}{c_2} \), its velocity \( c_2 = \sqrt{\frac{\mu}{\rho}} \), \( \lambda \) and \( \mu \) are the Lamé constants of the material, and \( \rho \) is its density.

The solution of the problem for an incident wave is trivial [9]; therefore, we shall examine the problem for the waves reflected from the cracks, which, as was shown in [3, 4, 6], is described by a steady periodic rather than a harmonic process. The contact forces produced by deformation on the crack edges \( q ( x , t ) \) and the displacement-discontinuity vector \( \Delta u ( x , t ) = \Delta u^+ ( x , t ) - \Delta u^- ( x , t ) \), where \( \Delta u^+ ( x , t ) \) and \( \Delta u^- ( x , t ) \) are the displacements of the opposite edges, must satisfy the unilateral-connection conditions formulated in [5], which lead to a nonlinear problem for the reflected waves, since the contact region of the crack edges changes with time and is unknown beforehand

\[
\Delta u_n ( x , t ) \leq h_\delta ( x ) , \quad q_n ( x , t ) \geq 0 , \quad ( \Delta u_n ( x , t ) + h_\delta ( x ) ) \cdot q_n ( x , t ) = 0 ,
\]

(3)
where the constants \( k_\tau \) and \( \lambda_\tau \) are determined by the properties of the contacting surfaces and \( h_0(\mathbf{x}) \) is the initial crack opening.

The constraints on the normal components of the vector of displacement discontinuity and the vector of contact forces are determined by the fact that the crack edges must not be “superposed” with deformation and the contact forces cannot be tensile. The constraints on the tangential components \( \Delta u_{\tau}(\mathbf{x}, t) \) and \( q_{\tau}(\mathbf{x}, t) \) are functions of the presence of friction in the contact region (we assume that it proceeds according to Coulomb’s law \([3]\)).

We expand the vector of the load on the crack edges and the displacement vector into Fourier series. Then, to determine the Fourier coefficients of the displacement vector, we have an infinite-dimensional system of differential equations, which follows from the equations of the linear dynamic theory of elasticity

\[
A_{ij} u_{ij}^k + \rho \cdot (\omega k)^2 u_{ij}^k = 0,
\]

\[
A_{ij} = \mu \cdot \delta_{ij} \cdot \delta_k + (\lambda + \mu) \cdot \partial_{ij} \partial_k, \quad k = 0, \pm 1, \pm 2, \ldots, \pm \infty.
\]

unilateral conditions (3) on the crack surface, and Sommerfeld’s conditions. The Fourier coefficients of the load vector have the form

\[
p^k(\mathbf{x}) = \begin{cases} q^k(\mathbf{x}), & \text{for all } k \neq 1, \\ \sigma_0(\mathbf{x}) + q^k(\mathbf{x}), & k = 1. \end{cases}
\]

where \( \sigma_0(\mathbf{x}) \) is the amplitude of the fictitious load on the crack edges caused by the incident wave.

The problem formulated for the reflected waves can be reduced to a system of integral boundary equations with homogeneous constraints. For this, it is sufficient to apply expansion into a Fourier series to a dynamic formula of the Somigliana type. Then we find that the Fourier coefficients of the contact-force vector and the displacement-discontinuity vector can be interrelated by an infinite-dimensional system of integral equations

\[
q^k(\mathbf{x}) = \begin{cases} \sigma_0(\mathbf{x}) - \frac{1}{\Omega} \int_{\Omega} F(\mathbf{y} - \mathbf{x}, k) \cdot \Delta u^k(\mathbf{y}) \cdot d s_y, & k = 1; \\ - \frac{1}{\Omega} \int_{\Omega} F(\mathbf{y} - \mathbf{x}, k) \cdot \Delta u^k(\mathbf{y}) \cdot d s_y, & k \neq 1, \end{cases}
\]

where the kernels in the integral representations are the Fourier coefficients of the Green functions of dynamic elasticity theory \([6]\), where, as was shown in \([2, 3]\), \( F(\mathbf{y} - \mathbf{x}, k) \rightarrow r^{-2}, y \rightarrow x \), i.e., the potential with the kernel \( F(\mathbf{y} - \mathbf{x}, k) \) has a “strong” singularity and must be considered in the sense of the Hadamard finite part \([1, 7]\). The problem for the reflected waves was thus reduced to solution of system of integral boundary equations (4) with allowance for constraints (3).

We write Eqs. (6) in a local coordinate system; the \( x \) axis is directed along the crack and the \( y \) axis is normal to its contour. Then the stresses on the crack edges caused by an incident compression wave are determined by the formulas

\[
\sigma_0(x) = \frac{k^2 \cdot \phi_{1 \cdot T}}{2} \left( \begin{array}{c} \sin(2\beta) \\ \lambda + 2\mu \sin^2(\beta) \end{array} \right) \cdot e^{i k_1 (V_1 \cos \beta)}, \quad x \in \Omega_1;
\]

\[
\sin(2\beta - 2\alpha) \\ \lambda + 2\mu \sin^2(\beta - \alpha) \end{array} \right) \cdot e^{i k_1 ((V_1 + h_1) \cdot \cos(\beta - \alpha) + (h_2 - h_2) \cdot \sin(\beta - \alpha))}, \quad x \in \Omega_2.
\]

For points \( y \) and \( x \) that belong to the same crack, \( F_{12}(\mathbf{y} - \mathbf{x}, k) = F_{21}(\mathbf{y} - \mathbf{x}, k) \equiv 0 \), while \( F_{11}(\mathbf{y} - \mathbf{x}, k) \) and \( F_{22}(\mathbf{y} - \mathbf{x}, k) \) are determined by the formulas