TRANSIENTS IN A TWO-PHASE MEDIUM FLOWING IN A PIPELINE

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The paper examines a water hammer in an elastic pipeline through which a liquid containing solid particles flows. The effect of the particle phase on the pressure jump in the liquid–particle mixture is studied.

N. E. Zhukovskii [3] was the first to study water hammer in pipelines. This study has initiated investigations of the transient pressure flow of a homogeneous liquid in pipelines. In the present paper, we study water hammer in a pipeline with a flowing two-phase medium (a particulate liquid).

We consider a semiinfinite pipeline with the beginning at x = 0. Let the Ox axis be directed against the stream of the two-phase medium. We assume that the law governing the change in mixture flow at the cross-section x = 0 is given. To describe the two-phase medium, we use the equations of gas dynamics of interpenetrating motions of compressible media [4]. According to [1, 4], a continuous “medium” of particles is introduced alongside with the carrying agent (the liquid). Apart from the true values of the densities \( \rho_1 \) and \( \rho_2 \), respectively, of the carrying agent and the “medium” of particles at each point of the volume occupied by the mixture, we also consider the average densities \( \gamma_1 \) and \( \gamma_2 \), which are defined as the mass of the particles per unit volume of the mixture. The average densities are related to the true values as \( \gamma_i = \alpha_i \rho_i \) (i = 1, 2), where \( \alpha_i \) are the concentrations of the phases. At each point of the mixture, we also introduce the velocity vectors \( v_1 \) and \( v_2 \) of the carrying agent and the particle phase, respectively. Both media are ideal, and the pressure at each point is assumed common for both components of the medium [4]. Let the particles be spheres of radius \( a \). The particles are considered so small that gravity does not cause them and the liquid to move relative to each other. The elasticity of the walls of the pipeline is taken into account according to [3].

Considering the foregoing, we write the system of equations describing the motion of the two-phase medium in the one-dimensional case. The equations of continuity are

\[
\frac{\partial (\gamma_i \sigma^2)}{\partial t} - \frac{\partial (\gamma_i \sigma^2)}{\partial x} = 0,
\]

where \( \sigma \) is the cross-sectional area of the pipeline.

The equations of motion are

\[
\frac{\partial v_i}{\partial t} = -\frac{1}{\rho_i} \frac{\partial P}{\partial x} + (-1) \frac{K_{ij}}{\gamma_i} (i = 1, 2, \quad j = 3 - i),
\]

where \( K_{ij} \) is the coefficient of the interphase interaction.

According to [4], the coefficient \( K_{ij} \) is the sum of two terms. The first term is associated with drag according to the Stokes law, and second term is due to the effect of added mass. Thus, for \( K_{ij} \), we obtain the expression

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\[ K_{ij} = - K_{ij} = \frac{9 \nu \rho_1 \gamma_2}{2 a^2 \rho_2} (v_2 - v_1) + \frac{\gamma_2 \rho_1}{2 \rho_2} \partial_t (v_2 - v_1). \]

The constitutive equations for the liquids and the solid phase is

\[ P = P_i (\rho_i, c_i), \]

where \( c_i \) is the sound speed.

The concentrations of the liquid phase and the particle phase are related as

\[ \frac{\gamma_1}{\rho_1} + \frac{\gamma_2}{\rho_2} = 1. \]

The elasticity of the pipeline is taken into account according to [3]

\[ P = \frac{h E}{R_0^2} R + h \bar{p} \frac{d^2 R}{dt^2}, \]

where \( h \) is the cross-sectional area of the pipeline; \( E \) and \( \bar{p} \) are the elastic modulus and the density of the material of the pipeline; \( R \) is the inner radius of the pipeline; and \( R_0 \) is the initial value of \( R \).

We linearize the obtained system in the neighborhood of the quiescent state

\[ P = P_0 + P', \quad \rho_i = \rho_{i0} + \rho_i', \quad \gamma_i = \gamma_{i0} + \gamma_i', \]
\[ v_i = v_{i0} + v_i', \quad R = R_0 + R'. \]

Here, \( P_0, \rho_{i0}, \gamma_{i0}, v_{i0}, \) and \( R_0 \) are the initial values of the corresponding quantities. The variables with primes (perturbations) are small in comparison with the initial values. Then, we obtain the following system for the perturbations (the primes are omitted):

\[ \frac{\gamma_1}{\rho_{10}} \frac{\partial v_1}{\partial x} - \frac{1}{\rho_{10}} \frac{\partial P}{\partial x} - \frac{h_1}{\gamma_{10}} \left( v_2 - v_1 \right) - \frac{h_2}{\gamma_{10}} \frac{\partial}{\partial t} \left( v_2 - v_1 \right) = 0, \]
\[ \frac{\gamma_2}{\rho_{20}} \frac{\partial v_2}{\partial x} - \frac{1}{\rho_{20}} \frac{\partial P}{\partial x} + \frac{h_1}{\gamma_{20}} \left( v_3 - v_1 \right) + \frac{h_2}{\gamma_{20}} \frac{\partial}{\partial t} \left( v_2 - v_1 \right) = 0, \]
\[ \frac{\gamma_1}{\rho_{10}} + \frac{\gamma_2}{\rho_{20}} = \lambda P, \quad P = \frac{h E}{R_0^2} R + h \bar{p} \frac{\partial^2 R}{\partial t^2}. \]

Here,

\[ h_1 = \frac{9 \nu \alpha_2}{2 a^2 \rho_{10}}, \quad h_2 = \frac{\alpha_2}{2 \rho_{10}}, \quad \lambda = \frac{\alpha_1}{\rho_{10} c_1^2} + \frac{\alpha_2}{\rho_{20} c_2^2}. \]

We transform the system of equations obtained. Subtracting the third equation in (1) from the fourth one and introducing the notation

\[ v_2 - v_1 = u_1, \quad \frac{1}{\rho_{20}} - \frac{1}{\rho_{10}} = \lambda_2, \quad \frac{1}{\gamma_{20}} = \frac{1}{\gamma_{10}} = \lambda_3, \]