OPTIMAL QUASI-CIRCULAR MOTION OF A SPACECRAFT
WITH AN ENERGY STORAGE DEVICE

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The problem of using an energy storage device in spacecraft low-thrust propulsion systems with a constant-power thruster is considered and solved for optimal quasi-circular maneuvers in the near-Earth space. The maximum payload is the optimality criterion. The optimal control as a function of time and the optimal mass parameters of the spacecraft were determined. Domains of the mass parameters where the use of an energy storage device makes sense are shown.

1. Kiforenko [4] studied the plane problem of motion of a spacecraft (SC) with a low-thrust propulsion system, a constant-power thruster, and an energy storage device. The radial component of the SC acceleration due to thrust is assumed to be equal to zero.

We will consider the three-dimensional problem without the above assumption. Let

\[ M_y = \gamma N_y, \quad M_v = \alpha N_0, \quad M_c = \beta E_0, \]

where \( M_y \) is the mass of the thruster; \( M_v \) is the mass of the power source; \( M_c \) is the mass of the energy storage device; \( N_y \) is the power consumed by the thruster; \( N_0 \) is the maximum power of the source; \( E_0 \) is the power capacity of the energy storage device; \( \alpha, \beta \), and \( \gamma \) are constants of proportionality.

We neglect all the external forces except for the Earth gravity. Then in the equatorial spherical coordinate system, the equation of the SC motion in the near-Earth space along quasi-circular orbits has the form [1]

\[ \ddot{r} - r \dot{\theta}^2 \cos^2 \theta - r \dot{\phi}^2 + 1/r^2 = a_r, \]
\[ r \cos \theta \dot{\phi} + 2 \dot{r} \cos \theta - 2 \dot{r} \sin \theta = a_{\phi}, \]
\[ r \ddot{\theta} + 2 \dot{r} \dot{\phi} + r \dot{\phi}^2 \sin \theta \cos \theta = a_{\theta}. \]

Here, we neglect perturbations due to the Earth's nonsphericity, since they are values of higher order of smallness as compared to those taken into account in (1.2) [2]. We also write the initial and final conditions in a spherical coordinate system. Equations (1.2) are dimensionless. In these equations, \( r \) is the distance to the attractive center referred to the radius of the initial circular orbit, and \( a_r, a_{\phi}, \) and \( a_{\theta} \) are the projections of the accelerations due to thrust referred to the gravitational acceleration at the distance of the initial radius. The time is referred to the period of revolution along a circular orbit of initial radius divided by \( 2\pi \).

The change in the power level in the energy storage device is described by the equation [2]

\[ \dot{e} = -\frac{\eta}{\zeta_b} (N_y \delta - N), \]

\[ \frac{1}{2} \]
where \( \xi_b = \frac{M_r}{M_v} \), \( \eta = \frac{\beta T^*}{\alpha} \); \( N_\gamma \) and \( N \) are the thruster power and the current power of the source, both referred to \( N_\mu \); \( T^* \) is the period of revolution along a circular orbit of initial radius divided by \( 2\pi \); and \( \delta \) is a controlling function (\( \delta = 0 \) for passive trajectory phases and \( \delta = 1 \) for active ones).

The problem on the payload mass maximum is reduced to minimization of the functional \( J_c \). The derivative of this functional has the form [2]

\[
J_c = \left( \frac{1 + \xi_b}{N_\gamma} + \varepsilon \right) (a_\xi^2 + a_\mu^2 + a_\nu^2) \delta,
\]

where

\[
\varepsilon = \frac{\chi}{\alpha}.
\]

To derive a complete variational problem in Mayer's form, it is necessary to add to Eqs. (1.3) and (1.4) the equation of SC motion (1.2) with the corresponding initial and final conditions.

2. Let us consider the problem on transferring an SC with maximum payload mass from the initial orbit to the new one in a half-revolution. The new orbit has radius \( r_1 \) and inclination \( i \), and \( |r_1 - 1| < 1 \), \( |i| < 1 \). This problem is the basis for problems on major changes in the orbit parameters and interplanetary cruises [3]. Let us consider the new variables \( !u = r - 1 \), \( \tilde{\xi} = \mu - \varphi \), and \( \zeta = \theta \). Then \( |\mu| < 1, |\xi| < 1 \), and \( |\zeta| < 1 \). Linearizing system (1.2) and using formulas (1.3) - (1.4) and the initial and final conditions from [2], we write the mathematical statement of the problem

\[
J_c = \left( \frac{1 + \xi_b}{N_\gamma} + \varepsilon \right) (a_\xi^2 + a_\mu^2 + a_\nu^2) \delta, \quad J_c (0) = 0, \quad J_c (\pi) = \min.
\]

In the Mayer problem (2.1), the number of equations is less than that of the unknown functions. The functions \( a_\xi, a_\mu, a_\nu, \) and \( \delta \) represent controls, which are to be determined using boundary conditions at the final point of the phase space. This is a problem with phase constraints on the coordinate \( e \). According to Pontryagin's maximum principle and the theorems proved in [5], the cases \( 0 < e < 1 \) and \( e = 0, e = 1 \) are considered separately. Let us write the function \( H \) for the case \( 0 < e < 1 \):

\[
H = - \left( \frac{1 + \xi_b}{N_\gamma} + \varepsilon \right) (a_\xi^2 + a_\mu^2 + a_\nu^2) \delta - \Psi_\xi \frac{\eta}{\xi_b} (N_\gamma \delta - N) + \Psi_\xi u + \Psi_\mu \nu + \Psi_\nu w.
\]

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