THE ITERATIVE THEORY OF DEFORMATION OF LAMINATED SHELLS

A. V. Plekhanov

The equations of the iteration theory of nonshallow transversally isotropic laminated shells, which account for all the components of the stress–strain state (SSS) and describe the inner SSS, potential, and vortex boundary effects, are obtained. The equations are based on the method of expansion of SSS into series in transverse coordinate and the method of variation with respect to the state being determined. The order of the equations does not depend on the number of layers and expansion terms that approximate the displacement and stress. The accuracy of the solution for the inner SSS and boundary effects is estimated.

An analysis of the well-known theories of laminated shells and plates shows that the majority of them describe with a certain accuracy, as a rule, only the inner stress–strain state (SSS) corresponding to integral boundary conditions. If some of them describe edge effects of boundary-layer type (potential and vortex edge effects), then they do this only as a first approximation, with insufficient accuracy, or the realization of the equations obtained involves great difficulties.

In the present study, the method of expansion of SSS components into series in terms of functions of transverse coordinates is used in combination with the method of variation with respect to the state being determined [2, 4] to derive equations of the iterative theory of nonshallow laminated transversally isotropic shells. The order of these equations does not depend on the number of retained expansion terms, which allows us to construct solutions of applied problems in high approximations. The equations allow for all of the SSS components and describe both the inner SSS and edge effects of boundary-layer type. The order of the equations does not depend on the number of layers.

Let us consider a laminated shell of constant thickness \( h = h_1 + h_2 \), consisting of an arbitrary number \( m \) of elastic transversally isotropic layers of thickness \( t_k \) (\( k = 1, 2, ..., m \) is the layer number reckoned from the lower to the upper layer). The coordinate surface \( x_3 = 0 \), located at distance \( h_1 \) [3] from the lower front face of the shell, is referred to an orthogonal system of curvilinear coordinates \( x_1 \) and \( x_2 \) corresponding to the lines of principal curvatures of this surface. We will represent a transverse load on the front faces of the shell in the form of symmetric \( p \ (x_1, x_2) \) and skew-symmetric \( q \ (x_1, x_2) \) components so that for \( x_3 = h_1 - h_2 \) we have

\[
\sigma_3 = 0.5 \ (-p \pm q), \quad \sigma_{13} = \sigma_{23} = 0. \tag{1}
\]

To reduce the three-dimensional problem of the theory of elasticity to a two-dimensional one, we will use the method of expansion of the components of stress and displacement into series in terms of functions of transverse coordinate and assume that [3]

\[
\sigma_1^k = \frac{E_0}{B} f_0(x_3) N_1(x_1, x_2) + \frac{E_0}{D_1} \sum_{i=1}^{\infty} f_i(x_3) M_1^i(x_1, x_2) \quad (1 \to 2.12),
\]
The expansion terms with indices \( i = 0, 1 \) describe stress states that are not self-balanced across the thickness of the shell (the first state), and the terms with indices \( i \geq 2 \) describe self-balanced states; \( N_1 \) and \( N_2 \) are normal forces, and \( N_{12} \) is shear force; \( M_1 \) and \( M_2 \) are bending moments, and \( M_{12} \) is torque; \( Q_1 \) and \( Q_2 \) are transverse forces; \( M_{11} \), \( M_{12} \), \( Q_{12} \), \( Q_{11} \), and \( \omega^i \) (\( i \geq 2 \)) are polymoments and polyforces corresponding to the stresses \( \sigma_1, \sigma_2, \sigma_{12}, \sigma_{13}, \sigma_{23}, \) and \( \sigma_3 \) self-balanced across the thickness of the shell.

Approximating functions (2) satisfy (1) and the conditions of tight contact of layers and possess the properties of completeness and linear independence.

To derive equilibrium equations, boundary conditions, and elasticity relations, we will use the mixed Reissner variation principle in combination with the state-variation method. Such an approach, as opposed to the well-known ones, allows us to obtain a sequence of differential equations whose order does not depend on the number of terms of expansions (2), approximating displacements and stresses. As a result, an iterative process obtains, for which the functions of the previous states appear as knowns in the equation for the subsequent states, playing the role of load terms. In this case, the SSS of a laminated shell to a certain approximation represents a combination of the first (nonself-balanced) state and an appropriate number of the self-balanced states. It is important to note that the state-variation method is valid if stress states, beginning from the second one, are self-balanced across the thickness of the shell. Approximating functions taken in the form of (2) satisfy this condition. The convergence and accuracy of solutions for the inner SSS and edge effects of boundary-layer type are considered in [15] on the basis of the state-variation method.

Following the state-variation method, from the Reissner variation equation, we obtain the equilibrium equation the first (nonself-balanced) state (\( i = 0, 1 \))

\[
(A_2 N_1)_{,1} + (A_1 N_{12})_{,2} - A_{2,1} N_2 + A_{1,2} N_{12} - A_1 A_2 k_1 Q_{1,1} = 0 \quad (1 \leq 2),
\]

\[
(A_2 M_{11})_{,1} + (A_1 M_{12})_{,2} - A_{2,1} M_1 + A_{1,2} M_{12} - A_1 A_2 k_1 Q_{1,1} = 0 \quad (1 \leq 2),
\]

\[
(A_2 Q_{11})_{,1} + (A_1 Q_{12})_{,2} + A_1 A_2 (k_1 N_1 + k_2 N_2 + q) = 0;
\]

the \( j \)th self-balanced state (\( j \geq 2 \))

\[
L_{1ji} [(A_2 M_{11})_{,1} + (A_1 M_{12})_{,2} - A_{2,1} M_1 + A_{1,2} M_{12}] - \sum_{i=1}^{j} (L_{3ji} + k_1 L_{310j}) A_1 A_2 Q_{1,1} = 0 \quad (1 \leq 2),
\]

\[
\sum_{i=1}^{j} L_{3ji} [(A_2 Q_{11})_{,1} + (A_1 Q_{12})_{,2}] + \frac{D_1}{B} L_{130j} A_2 (k_1 N_1 + k_2 N_2)
\]

\[
+ \sum_{i=1}^{j} L_{130j} A_1 A_2 (k_1 M_{11} + k_2 M_{12}) + 0.5 L_{20j} A_1 A_2 p
\]

\[
+ \sum_{i=1}^{j} L_{20j} A_1 A_2 \omega^i + 0.5 A_1 A_2 (T_{1ji} q - T_{2ji} p) = 0.
\]