DESIGN OF A HEATER FOR DRYING ARTICLES
MADE OF CURRENT-CONDUCTING MATERIALS

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We present a description and calculation of the temperature and moisture-content fields of capillary-porous materials in the process of heating them by an electric current with removal of hygroscopic moisture.

Use a volumetric method of heat supply considerably increases the efficiency of heat treatment of articles, including drying. The electrical-conduction properties of some materials allow heating them without applying secondary heat sources. The temperature difference over the specimen cross section appears in the course of electric heating, favors transfer of moisture from its central layers to the periphery [1, p. 319]. When the heating is sufficiently intense, a filtrational flux appears that is attributable to the total-pressure gradient and is likewise directed to the surface. All this increases the economic performance figures of drying and is indicative of the prospects for using the electrical-contact method of heating [2].

The distribution of moisture in an article being treated is determined by the temperature field, for whose description the Fourier differential equation of heat conduction is used [3].

If we neglect the influence of heat losses in contacts on the temperature distribution, we can write the heat-balance equation as

$$ mC \frac{dt}{dt} + \alpha F (t - t_0) = A l^2 R. $$

(1)

Since the motion of the bound substance in a capillary-porous body is considered to be rather slow, the temperature of the liquid is virtually equal to that of the walls of the capillary [4]. Many parameters change in the course of heating: current, the resistance of the article, moisture content, thermal-conductivity coefficient, the heat capacity of the moist material, etc. It is virtually impossible to relate all these parameters to the temperature of the article, and therefore we divide the heating curve into a number of intervals within which the values of I, R, U, λ, C will be considered constant. Separating variables and integrating Eq. (1), we determine the time of heating the article for any calculational interval:

$$ \tau_i = \frac{mCL_i}{\alpha F} \ln \frac{A l^2 R - \alpha F (t_{\text{end}} - t_0)}{A l^2 R - \alpha F (t_{\text{beg}} - t_0)}. $$

(2)

The total time of heating is $\tau = \sum_{i=0}^{n} \tau_i$.

With consideration of the aforesaid, the differential equation of heat conduction in the case of the one-dimensional problem takes the form [3]

$$ \frac{\partial t}{\partial \tau} = \frac{\partial^2 t}{\partial x^2} + \frac{I}{x} \frac{\partial t}{\partial x} + \frac{q}{C \rho_0}. $$

(3)
We will consider the process of heating a cylindrical article whose length greatly exceeds its radius; in this case the geometric parameter \( \Gamma \) is equal to unity.

The initial conditions are: the temperature over the entire cross section is the same and equal to

\[
|f|_{r=0} = t_{\text{beg}}.
\]

The boundary conditions are: 1) boundary conditions of the third kind on the body surface:

\[
- \lambda \frac{dt}{dr} \bigg|_{r=r_0} = \alpha (t - t_0) + \varepsilon r^* t_{\text{sur}}^* + C_{\text{vap}} t_{\text{sur}}^* t_{\text{sur}}^* ,
\]

where \( \alpha = \alpha_{\text{rad}} + \alpha_{\text{conv}} \); \( \alpha_{\text{rad}} \) can be calculated as [1]

\[
\alpha_{\text{rad}} = \frac{\varepsilon_{\text{red}} \left[ \left( \frac{T}{100} \right)^4 - \left( \frac{T_0}{100} \right)^4 \right]}{t - t_0} ;
\]

values of \( \alpha_{\text{conv}} \) are given in [2, p. 23]; 2) the solution is bounded along the article axis \( (r = 0) \).

In passing a current through a cylindrical billet, the volumetric density of internal heat sources is distributed according to the law

\[
q = \frac{A \mu_{\text{mat}} \mu_0}{4\pi r_0^2 J_0 (\xi_{\text{sur}} r_0)} \left| J_0 (\xi_{\text{sur}} r_0) \right|^2 ,
\]

where \( \xi_{\text{1}} = \sqrt{-\frac{\lambda}{\mu_{\text{mat}} \mu_0}} \).

Let \( \alpha r / r_0^2 = \phi_0 \), \( r / r_0 = z \), \( t - t_0 = \theta \); then Eq. (3) can be rewritten in a more convenient form:

\[
\frac{\partial \theta}{\partial \phi_0} = \frac{\partial^2 \theta}{\partial z^2} + \frac{1}{z} \frac{\partial \theta}{\partial z} + k_1 \left| J_0 (\xi_{\text{1}} z) \right|^2 ,
\]

where

\[
k_1 = \frac{A \mu_{\text{mat}} \mu_0}{4\pi \lambda^2 J_0 (\xi_{\text{1}})} ; \quad \xi_{\text{1}} = \frac{\phi_0 \sqrt{-\frac{\lambda}{\mu_{\text{mat}} \mu_0}}}{r_0} ,
\]

with the initial conditions

\[
|\theta|_{\phi_0=0} = \theta_{\text{beg}}
\]

and the boundary conditions:

1) \( - \lambda \frac{d\theta}{dz} \bigg|_{z=1} = \alpha r_0 \theta + \varepsilon r^* t_{\text{sur}}^* + C_{\text{vap}} r_0^* t_{\text{sur}}^* \theta ,
\]

2) \( z = 0 \) -- the solution is bounded.

The solution of Eq. (8) with initial and boundary conditions (9)-(11) is obtained in the form

\[
\theta = \frac{4k_1}{q_1} \left( \frac{1}{B \nu_1 + \chi_1 + \chi_2} \right) + \sum_{n=1}^{\infty} C_{\text{vap}} J_0 (\mu_n z) \exp (-\mu_n^2 \phi_0) =
\]