ENGINEERING CALCULATION OF THE CHARACTERISTICS OF A FILTRATION-COMBUSTION WAVE BASED ON A ONE-DIMENSIONAL TWO-TEMPERATURE MODEL

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The authors present a method for determining the characteristics of a filtration-combustion wave that uses physically substantiated simplifications of a one-dimensional two-temperature model of filtration flow with Arrhenius kinetics. Explicit, physically transparent dependences for the ignition temperature, wave velocity, and maximum skeleton temperature are obtained. A brief review of and a comparison with other one-dimensional methods and with a numerical calculation are given. This method is shown to be superior to more cumbersome analytical two-temperature methods in accuracy.

The process of combustion in a porous medium, or filtration combustion, attracts great interest of researchers in connection with numerous technical applications of this process such as recycling (oxidation) of noxious gases, combustion of lean fuel mixtures in a superadiabatic-combustion regime, thermal-wave treatment of catalysts, and others [1, 2].

When real devices with FC are developed, understanding of the occurring processes and accessible apparatus for calculating the basic characteristics of FC waves are required foremost. The majority of works on FC of gases, on the other hand, contain cumbersome formulas of little use for engineering calculations. In the present work, an attempt is made to provide a more general picture of the methods of analysis of one-dimensional stationary FC of gases in an inert skeleton, and a simple explicit formula to calculate the ignition temperature and the maximum skeleton temperature is derived. This approach is based on physically substantiated simplifications of a one-dimensional two-temperature model of filtration combustion with Arrhenius kinetics. Problems of unsteadiness and wave stability and other more complicated issues are not considered in this paper.

In [3, 11], regimes of slow filtration combustion and rapid (in fact, break of the flame through the pore space) filtration combustion are distinguished. By FC we will mean a slow-combustion regime, where intense interphase heat transfer governs the basic regularities of combustion-wave propagation.

The issues of analytical and numerical descriptions of filtration-combustion processes have been the objective of numerous works, references to which can be found in [2-4] and other works. In analysis and mathematical modeling, in principle, there can be two approaches to the statement and solution of the problem:

1) volume-averaged modeling, in which use is made of the effective (averaged over the volume) thermophysical characteristics of the porous skeleton and the gas, and the interphase interaction is described by the heat transfer coefficient α;

2) direct point modeling, in which the energy and momentum equations (for the gas) are considered and the boundary conditions are established at the interface.

Direct point modeling was investigated in the works of Kaviani et al. [4, 5]. As a result it was shown that this approach has no advantages over the volume-averaged approach when FC macroscopic parameters (structure, thickness, velocity, characteristic temperatures of the FC wave) are determined, although it predicts intrapore variations in the gas temperature up to 40% and in the combustion rate up to 20% [5]. Since results on intrapore thermal processes, in practice, are not checkable experimentally, it is unreasonable to use direct point modeling to...
investigate FC. An important drawback of it is computational tediousness and the impossibility of considering the FC problem successively and analytically. Therefore, in the majority of cases, the volume-averaged approach is used.

Methods for Analyzing One-Dimensional Problems of FC of Gases. In considering FC of gases, we usually solve a system of balance equations for the porous-body temperature, the gas temperature, and the concentration of the deficient reagent:

\[
(\rho\alpha) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) - \alpha (T - T_g) - \beta (T - T_0), \tag{1}
\]

\[
(\rho\alpha) \frac{\partial T_g}{\partial t} + (\rho\alpha) u_g \frac{\partial T_g}{\partial x} = \frac{\partial}{\partial x} \left( \lambda_g \frac{\partial T_g}{\partial x} \right) + \alpha (T - T_g) + QW(y, T_g), \tag{2}
\]

\[
\frac{\partial y}{\partial t} + u_g \frac{\partial y}{\partial x} = \frac{\partial}{\partial x} D \frac{\partial y}{\partial x} - W(y, T_g). \tag{3}
\]

Here the basic notation is standard and is given at the end of the paper. The subscript g denotes the gas component; the solid-phase parameters are without subscripts. The value of the porosity \( \eta \) is involved implicitly via the heat capacities, thermal conductivities, and heat-transfer coefficient.

The possibilities of analytical solution of such systems are limited, although important qualitative and quantitative relations can be obtained on the basis of certain model assumptions. Thus, considering the energy balance of the FC wave, we can show that the maximum equilibrium temperature of the steadily propagating wave in the absence of heat losses is related to this wave’s velocity:

\[
\Delta T_{\text{max}} = \frac{\Delta T_{\text{ad}}}{1 - \frac{u_w}{u_i}}, \tag{4}
\]

where \( \Delta T_{\text{max}} \) is the maximum (equilibrium with the gas) temperature in the combustion wave; \( \Delta T_{\text{ad}} \) is the adiabatic temperature of fuel-mixture combustion; \( u_i \) is the wave velocity in the absence of a reaction of heat release, determined as \( u_i = u_g(c\rho)_g/c\rho \); \( u_w \) is the actual velocity of the FC front. From (4), it can be seen that \( \Delta T_{\text{max}} \) can substantially exceed the adiabatic combustion temperature in the case of co-motion of the combustion wave and is always lower than \( \Delta T_{\text{ad}} \) in the case of countermotion of the wave. However without detailing the mechanisms of heat transfer and heat release it is impossible to determine important FC parameters such as \( u_w, \Delta T_{\text{max}}, \) and others.

In analyzing FC, wide acceptance has been received by a one-temperature approximation in which the gas \( T_g \) and solid-phase \( T \) temperatures are assumed to be equal due to the high intensity of the interphase heat transfer, \( \alpha \rightarrow \infty \). The two energy-balance equations (1) and (2) are reduced to one:

\[
(\rho (c\alpha) + (\rho\alpha) g) \frac{\partial T}{\partial t} + u_g (\rho\alpha) g \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left( \lambda + \lambda_g \right) \frac{\partial T}{\partial x} - \beta (T - T_0) + QW(y, T). \tag{5}
\]

If the FC wave propagates with a constant velocity \( u_w \), then in a coordinate system that moves with the wave, Eq. (5) takes the form

\[
- u_w (\rho (c\alpha) + (\rho\alpha) g) \frac{\partial T}{\partial x} + u_g (\rho\alpha) g \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left( \lambda + \lambda_g \right) \frac{\partial T}{\partial x} - \beta (T - T_0) + QW(y, T), \tag{6}
\]

or, by introducing the thermal velocity of the front \( u_i = u_g(c\rho)_g/c\rho \) and the dimensionless velocity of the front of the combustion wave \( u = u_w/u_i \), we have