RISE OF AN AIR BUBBLE (A THERMAL) IN THE ATMOSPHERE

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We elucidate the behavior of large regions of warm air (thermals) rising in the earth's atmosphere. It is found that inside a uniformly rising large spherical bubble rotational motion of the air arises around a central vortex line located in the equatorial plane of the bubble. We determine the total energy of the rise process of a thermal.

The rise of large bubbles (thermals) is one of the most common forms of convective motion in the terrestrial atmosphere [1-4]. Such a bubble contains warmer air compared to the surrounding atmosphere, and therefore an Archimedes force acts on it and causes its rise. This process is accompanied by compensating motion of neighboring layers of the atmosphere downward. It is clear that between the surface of the rising bubble and the descending layers of the surrounding atmosphere the forces of viscous friction appear whose magnitude increases with increase in the velocity of the bubble rise. Therefore, the friction forces turn out to be equal to the Archimedes forces already in the initial stage of the process of bubble rise, and the motion of the bubble becomes uniform, so that the medium is considered to be ideal.

We consider a three-dimensional idealized model of a thermal.

Since the air density $\mu$ inside the bubble is not much lower than the density of the atmosphere at the same height, we approximately assume the medium to be incompressible and the densities inside and outside the bubble to be equal.

The motion of the air bubble, whose shape is assumed to be spherical, will be interpreted as a local process [5-7]. We consider the bubble itself to be the core of a process in which the air of the bubble does not mix with the surrounding air.

It is known [8] that when a solid sphere moves in a fluid, an excess pressure appears on its faces $A$ and $B$, and conversely, a deficit of pressure in its equatorial (middle) plane. This gives rise to forces that strive to deform the sphere, i.e., to compress it in the vertical direction and stretch it equatorially. However, this deformation does not occur because of the rigidity of the sphere.

The situation is different when an air sphere moves in the atmosphere. However, as is seen from experiments, no deformation occurs in this case too. The natural question suggests itself: what hinders the deformation of a rising air bubble? We can obtain an answer to this question if we take into account the fact that the boundary spherical layer of the bubble air, due to its motion relative to the external region of flow, experiences a friction force that generates a torque causing sliding of the boundary spherical layer in a direction opposite to the rise of the bubble. In exactly the same way, the friction forces between the boundary spherical layer and a deeper spherical layer generates similar sliding of this latter layer. This is the way in which deeper and deeper air layers inside the bubble are drawn into internal motion. As a result, all the air in the bubble starts to rotate: layers far from the polar axis $AB$ move in the direction of the external air flow past the bubble (i.e., downward), and layers near the axis in the bubble move along the direction of bubble rise (i.e., upward) (Fig. 2).

This internal rotational motion of the air in the bubble opposes the forces that strive to deform it. Thus, the air of the bubble participates simultaneously in two motions: translational upward and rotational inside the sphere.
Fig. 1. Motion of a solid sphere in a fluid.

Fig. 2. Streamlines for an air bubble moving in the atmosphere: a) vertical section through the axis; b) middle plane.

To make the picture of the rotational motion in the bubble clearer, we divide its circular equatorial cross section (whose radius is \( R \)) by a dashed circle into two parts with equal areas: a disk \( M \), whose radius is \( R/\sqrt{2} \), at the axis and a plane ring \( N \), whose width is \( R - r \). The areas of the both parts of the equatorial circle are equal to \( \pi R^2/2 \). In the ring \( N \) the air moves downward (in a coordinate system associated with the rising bubble), and in the disk \( M \) the lines of rotational motion are directed upward.

The circle \( L \) (represented in Fig. 2b by the dashed line) lying in the middle plane and dividing the regions \( M \) and \( N \) will be called the central vortex line. The air in the thermal rotates around this line. The remaining vortex lines are also circles with centers on the polar axis; they are located in all the planes perpendicular to the axis \( AB \), including the horizontal middle plane. From the symmetry of the problem it is clear that on each circular vortex line \( \text{rot} \, \mathbf{v} \) has the same value.

Thus, the air rotating in the bubble forms a closed vortex tube whose cross section is in the form of a semicircle (Fig. 2) and that curls around the polar axis without a gap and occupies the entire volume of the bubble.

It is only necessary to keep in mind that the surface layer of the air in the bubble comes into contact with a zone of external flow that moves without vortices. Therefore, it follows from the continuity condition that a monotonic decrease in \( |\text{rot} \, \mathbf{v}| \) should occur inside the bubble from the central vortex line to its surface.

We encounter a similar situation in the axial region, where all the rotating streams merge into one rectilinear flow directed upward along the axis \( AB \). This means that in the axial region, too, the angular velocity of air rotation decreases with distance from the central vortex line to the axis.

From what has been said it follows that inside the bubble the quantity \( |\text{rot} \, \mathbf{v}| \) is maximum in the vicinity of the central vortex line \( L \) and gradually decreases to zero with distance from the line to each side.

Now, we consider this problem quantitatively. For this purpose, we use well-known solutions for spatial flow around a sphere [8, 9].

In a fixed absolute reference frame the air bubble moves upward along the \( z \) axis with the velocity \( \mathbf{v}_0 \). In a frame of reference moving with the bubble, the surrounding air moves with the velocity \( \mathbf{v}_f = -\mathbf{v}_0 \). When a sphere