ULTRAFILTRATION IN A PIPE FILTER WITH GELATION

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We investigate the process of unsteady-state ultrafiltration with gelation under laminar flow conditions in a pipe filter with nonideal selectivity of its membrane.

As a promising method for cleaning, separation, and concentration of dissolved or disperse particles, ultrafiltration has found wide application in the food, pharmaceutical, textile, metal-working, and electronic industries and in biology and medicine [1]. Hollow-fiber and pipe membrane apparatuses have come into widespread use.

The process of ultrafiltration is accompanied by the phenomenon of concentration polarization, which occurs in a pre-gel regime in hollow-fiber filters and in a gel regime in pipe filters [2].

To describe concentration polarization, the literature usually resorts to an integral method [3, 4], whose drawbacks are discussed in [5]. In [6], using a semi-integral method [5], a description is given for concentration polarization in axisymmetric membrane elements (hollow fibers and pipes) in a pre-gel regime.

Below we consider the gel regime of polarization in continuous-flow laminar ultrafiltration with nonideal selectivity of the membrane in a pipe filter.

With allowance for gelation, we obtain the velocity distribution in a cylindrical channel. We assume that the flow at the channel inlet is fully developed. The flow rate of the fluid through the channel cross section considerably exceeds the flow leaving through the membrane, and the thickness of the gel layer is much smaller than the channel radius. Then the equations of motion and continuity take the form

\[ \hat{u}_{rr} + \frac{1}{r} \hat{u}_r = \frac{1}{\mu} \hat{p}_z, \]

\[ \hat{p}_z = 0, \]

\[ \hat{v}_r + \frac{\hat{v}_r}{r} = - \hat{u}_z. \]

under the following boundary conditions:

\[ \hat{u} = 0, \quad \hat{v} = V_\delta \quad (r = R - \delta); \]

\[ \hat{v} = 0, \quad \hat{u}_r = 0 \quad (r = 0). \]

From Eq. (2) it follows that \( p = p(z) \).

Integrating Eq. (1) with allowance for the first boundary condition (4), we obtain
The equation of continuity yields

$$\hat{v} = -\frac{R^3}{16\mu} \frac{\partial p}{\partial z} \left( \frac{R^2}{2} - \frac{(R - \delta)^2}{R^2} \right).$$

Hence

$$\hat{V}_\delta = \frac{R^3}{16\mu} \frac{\partial p}{\partial z} \left( 1 - \frac{\delta}{R} \right)^3. \tag{8}$$

The mean flow rate at the channel inlet with $z = 0$ is given by the expression

$$\bar{u}_0 = \frac{R^2}{8\mu} \left( \frac{-\partial p}{\partial z} \right). \tag{9}$$

Integrating Eq. (8) with account for relation (9) and substituting the result into Eq. (6), we obtain an expression for the longitudinal velocity component:

$$\hat{u} = \frac{2}{R} \left( \frac{\bar{u}_0 - 2 \delta V_\delta dz}{\left(1 - \frac{\delta}{R}\right)^3} \left( \frac{R - \delta}{R^2} - \frac{r^2}{R^2} \right) \right). \tag{10}$$

Now, we consider the equation of convective diffusion. It is taken into account that in ultrafiltration the thickness of the diffusive boundary layer $\Delta$ is much smaller than the pipe radius $R$. Introducing the new variable $y = R - r$ and retaining the principal terms, we obtain an equation of convective diffusion, which is written in a conservative form (here and below we will operate with the dimensionless quantities $u, v, V_\delta, \eta, \xi$):

$$\frac{\partial (\Theta - 1)}{\partial \tau} + \frac{\partial u (\Theta - 1)}{\partial \xi} - \frac{\partial v (\Theta - 1)}{\partial \eta} = \frac{1}{Pe} \frac{\partial^2 \Theta}{\partial \eta^2}. \tag{11}$$

Equation (11) describes the pre-gel and gel regimes of polarization but under different boundary conditions. The first regime exists from the channel inlet up to a certain point $\xi_1$, which is called the gelation point, where the concentration on the membrane attains the concentration of gelation $\Theta_\delta$. After the gelation point downstream the second regime of polarization is realized. As noted above, the pre-gel regime was analyzed in [6].

Now, we consider the gel regime.

The boundary conditions after the gelation point, $\xi \geq \xi_1$, take the form

$$u|_{\eta = \delta} = 0, \ v|_{\eta = \delta} = V_\delta, \tag{12}$$

$$\varphi V_\delta \Theta_\delta + \frac{1}{Pe} \Theta_\eta \bigg|_{\eta = \delta} = \Theta_\delta \delta_1, \tag{13}$$

$$\Theta|_{\eta = \Lambda} = 1, \ \Theta|_{\eta = \delta} = \Theta_\delta. \tag{14}$$

We relate the decrease in the permeability of the membrane to the thickness of the gel layer:

$$V_\delta = \frac{\nu}{1 + k\delta} . \tag{15}$$