COMPTON ATTENUATION COEFFICIENT IN SCATTERING BY MAXWELLIAN ELECTRONS

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Equations for the coefficient of attenuation of radiation due to Compton scattering by a thermal electron gas with an arbitrary temperature are summarized. A new representation is also obtained for the integral through which the attenuation coefficient, averaged over a relativistic, Maxwellian energy distribution of electrons, is expressed. This representation enables one to efficiently calculate the coefficient for a high-temperature electron gas. The accuracy of an approximate expression for the attenuation coefficient, corresponding to the assumption that scattering is isotropic in the rest frame of an electron, is also estimated.

1. Introduction

It is well known that Compton scattering plays a decisive role in forming spectra in the x-ray and gamma-ray ranges for a number of astrophysical objects such as active galactic nuclei, accretion disks around stars, and others in which there hot rarefied regions of gas with a high relative abundance of free electrons.

Although far from all the questions about the roles of various emission mechanisms in the observed spectra have been answered, many models of these objects have been constructed. Their spectra have been calculated within the frameworks of those models. Among recent papers we cite [1-3]. In all such calculations it is assumed that the optical depth of the regions in which scattering occurs is low. Such an assumption may be incorrect in many cases, however. The calculation of spectra for optically deep media requires extensive and cumbersome calculations because of the need to allow for strong variation of photon frequency during scattering. One must therefore have, in particular, equations for the coefficient of absorption (more precisely, of attenuation) that are accurate and convenient for programming, enabling one to calculate this quantity rapidly. The functions of redistribution in frequency and direction are also normalized to this coefficient, which enables one to monitor the calculation of those functions.

General equations for the Compton attenuation coefficient have been given in [4]. Calculating equations for averaging over a power-law electron energy distribution were derived in [5]. In the present paper we give equations for this coefficient for a Maxwellian momentum distribution of electrons with an arbitrary temperature. We first give the well-known equations for the case of a relativistic Maxwellian distribution. We then derive a new equation for the integral representing the averaged attenuation coefficient. This new equation enables one to rapidly calculate the coefficient for a high-temperature electron gas, when the average particle energy exceeds the rest mass of an electron. Exact values of the coefficient are also compared with its approximate representation corresponding to the assumption that scattering is isotropic in the electron's rest frame.
2. Averaging of a Cross Section over a Maxwellian Electron Distribution

In Compton scattering of radiation by a nondegenerate, thermal electron gas, the attenuation cross section, i.e., the coefficient of absorption calculated per electron in units of the Thomson cross section in accordance with [4], is determined from the equation

\[
\tilde{s}_0(x,y) = \frac{y}{4K_3(y)} \int e^{-\gamma y} \psi_{10}(xu) \frac{\mu \gamma + z}{\mu \gamma - z} \, du.
\]

(1)

Here \( x, \gamma, z = \sqrt{\gamma^2 - 1}, \) and \( y = mc^2/k_bT \) are the dimensionless radiation frequency, the electron energy and momentum, and the inverse temperature of the electron gas \( (k_b \) is the Boltzmann constant). One must perform double substitution inside the integral in (1) and take the difference between the results. We have the function

\[
\psi_{10}(\xi) = \frac{2}{\xi} \int_0^\xi s_0(\xi') d\xi'
\]

(2)

where \( s_0(\xi) \) is the total scattering cross section, for fixed momenta of the photon and electron, in units of the Thomson cross section, defined by the Klein-Nishina formula,

\[
s_0(\xi) = \frac{3}{8\pi^2} \left[ 4 + \left( \xi - \frac{2}{\xi} \right) \ln(1 + 2\xi) + 2\xi^2 \frac{1 + \xi}{(1 + 2\xi)^2} \right].
\]

(3)

A calculation of the integral in (2) yields the expression

\[
\psi_{10}(\xi) = \frac{3}{4\pi^2} \left[ \left( \xi + \frac{9}{2} + \frac{2}{\xi} \right) \ln(1 + 2\xi) - 4 - \xi + \frac{\xi^2}{1 + 2\xi} - 2g(\xi) \right],
\]

(4)

containing the nonelementary function

\[
g(\xi) = \frac{1}{\xi} \int_0^\xi \ln(1 + 2\xi') d\xi'.
\]

(5)

Integrating by parts in (1), we obtain

\[
\tilde{s}_0(x,y) = \frac{1}{2K_3(y)} \int e^{-\gamma y} \frac{d\xi}{z} \left[ (y + z)^2 s_0(x(y + z)) + (y - z)^2 s_0(x(y - z)) \right].
\]

(6)

These equations enable one to calculate \( \tilde{s}_0(x,y) \) for values of the arguments \( x \) and \( y \) on the order of unity. Power-law expansions can be used for low and high frequencies \( x \). We also give them.

3. Series Expansions

It is easy to show the correctness of the expansion

\[
s_0(\xi) = \sum_{n=0}^{\infty} a_n (-2\xi)^n,
\]

(7)

where

\[
a_n = \frac{3}{8} \left( \frac{n+2}{n+1} + \frac{8}{n+2} - \frac{16}{n+3} \right).
\]

(8)

The function \( \psi_{10}(\xi) \) can also be expanded in a power series,