NUMERICAL SIMULATION OF TIME-DOMAIN EM SCATTERING BY SHALLOW SUBSURFACE OBJECTS

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Abstract  The time-domain ElectroMagnetiC(EM) scattering by buried objects in dispersive media is calculated with FD-TD method. The FD-TD formula in Debye dispersive media (both the complex permeability and the complex permittivity are described by Debye equations) are deduced, and the absorbing boundary condition is given. The validity of FD-TD method in lossy media is verified through comparing the FD-TD's results and the other ones. The propagation of transient pulses in dispersive media is studied in detail. The scattering pulses and the wiggle traces for typical buried objects are given.

Key words  FD-TD-method; Dispersive media; Buried objects; Baseband pulse; Electromagnetic scattering

I. Introduction

Many methods developed for solving EM field problems in free space can not be applied to subsurface EM scattering because of the air/soil interface and the dispersivity of soil. The Finite-Difference Time-Domain (FD-TD) method put forward in 1960's has been widely used in calculating the EM problems in free space due to its simple and intuitive nature, its flexibility and ability to simulate many complex object structures [1,2]. The FD-TD method had been used to calculate the EM scattering of buried objects by us [3], but the homogeneity and nondispersivity of soil had been supposed, so it is obvious that the results obtained in that paper had bigger errors because many soils are inhomogeneous and dispersive under nanosecond pulse illuminating.

In this paper, the FD-TD method is extended to calculate the EM scattering by objects buried in half infinite medium space whose parameters are described by Debye model.

II. The FD-TD Method in Debye Media and Absorbing Boundary Condition

1. The formula of FD-TD method in Debye dispersive media

The parameters of the dispersive medium is assumed to have the forms of the Debye model with a single relaxation time, the equations of the Debye model in frequency-domain
can be described as:

\[
\varepsilon_r(\omega) = \varepsilon_{\infty} + \frac{\varepsilon_r - \varepsilon_{\infty}}{1 + j\omega\tau}, \quad \mu_r(\omega) = \mu_{\infty} + \frac{\mu_r - \mu_{\infty}}{1 + j\omega\tau} \tag{1}
\]

where \( \varepsilon_r \) and \( \varepsilon_{\infty} \) are the low- and high-frequency permittivities, \( \mu_r \) and \( \mu_{\infty} \) are low- and high-frequency permeabilities, respectively; \( \tau \) is the relaxation time, and \( \omega \) is the angular frequency.

The time-domain equations of Eq. (1) can be deduced as:

\[
\varepsilon_r(t) = \varepsilon_{\infty} \delta(t) + \frac{\varepsilon_r - \varepsilon_{\infty}}{\tau} e^{-t/\tau} U(t), \quad \mu_r(t) = \mu_{\infty} \delta(t) + \frac{\mu_r - \mu_{\infty}}{\tau} e^{-t/\tau} U(t) \tag{2}
\]

where \( \delta(t) \) and \( U(t) \) are the impulse function and unit-step function, respectively.

The components of the electric flux density \( \mathbf{D}(t) \) and their partial differential to time \( t \) in \( p \) direction (\( p = x, y, z \)) are

\[
D_p(t) = \varepsilon_0 \varepsilon_{\infty} E_p(t) + \frac{\varepsilon_0 (\varepsilon_r - \varepsilon_{\infty})}{\tau} \int_0^{+\infty} e^{-\frac{t-u}{\tau}} U(t-u) E_p(u) du \tag{3a}
\]

\[
\frac{\partial D_p(t)}{\partial t} = \varepsilon_0 \varepsilon_{\infty} \frac{\partial E_p(t)}{\partial t} + \frac{\varepsilon_0 (\varepsilon_r - \varepsilon_{\infty})}{\tau} \left[ E_p(t) - \frac{\Delta t}{\tau} S_p(t) \right] \tag{3b}
\]

\[
S_p(t) = \frac{1}{\Delta t} \int_0^{+\infty} e^{-\frac{t-u}{\tau}} U(t-u) E_p(u) du \tag{3c}
\]

Then the recurrence formula of \( S_p(\cdot) \) can be described as

\[
S_p(t) = e^{-\frac{At}{2}} S_p(t - \Delta t) + \frac{1}{2} [e^{-\frac{At}{2}} E_p(t - \Delta t) + E_p(t)] \tag{3d}
\]

where \( \varepsilon_0 \) is the permittivity of air.

It is obvious both \( D_p(\cdot) \) and \( E_p(\cdot) \) are the functions of space point \((x, y, z)\). The magnetic flux density \( \mathbf{B}(t) \) and its differential to time \( t \) can be deduced in a similar way.

In Cartesian coordinates, the formulas of the FD-TD method in Debye dispersive media can be derived in this way, putting \( D_p(t) \), \( B_p(t) \) and \( \partial D_p(t)/\partial t \), \( \partial B_p(t)/\partial t \) into the Eqs. (1a)–(1f) in Ref. [2] where are six scalar partial differential equations derived from Maxwell’s equations; positioning all of the components of the electromagnetic fields on Yee’s grids; replacing the space and time derivatives with centered finite-difference expressions. Then the formulas of the FD-TD method in Debye dispersive media can be developed through some deducing, for instance

\[
H_{x+1/2}(i, j + 1/2, k + 1/2) = \text{CAH}(m) \cdot H_{x-1/2}(i, j + 1/2, k + 1/2) + \text{CBH}(m) \cdot [\tilde{E}_y^n(i, j + 1/2, k + 1) - \tilde{E}_y^n(i, j + 1/2, k)] + \tilde{E}_x^n(i, j + 1/2, k + 1/2) - \tilde{E}_x^n(i, j + 1, k + 1/2) + \text{CCH}(m) \cdot W_{x-1/2}(i, j + 1/2, k + 1/2) \tag{4a}
\]