SEMI-BLIND CHANNEL ESTIMATION OF MULTIPLE-INPUT/MULTIPLE-OUTPUT SYSTEMS BASED ON MARKOV CHAIN MONTE CARLO METHODS

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Abstract This paper addresses the issues of channel estimation in a Multiple-Input/Multiple-Output (MIMO) system. Markov Chain Monte Carlo (MCMC) method is employed to jointly estimate the Channel State Information (CSI) and the transmitted signals. The deduced algorithms can work well under circumstances of low Signal-to-Noise Ratio (SNR). Simulation results are presented to demonstrate their effectiveness.

Key words Multiple-Input/Multiple-Output (MIMO) system; Channel estimation; Markov Chain Monte Carlo (MCMC) method

I. Introduction

It has been known that the capacity of wireless systems can be greatly increased by using multiple antennas at both transmitter and receiver. There is a linear growth in capacity proportional to the minimum number of transmitting and receiving antennas in the Multiple-Input/Multiple-Output (MIMO) system in the presence of multi-path scattering[1-3]. Some techniques such as BLAST[4] have been developed to achieve this capacity.

In MIMO systems, the Channel State Information (CSI) should be estimated for the detection of signal. Usually some pilot symbols known to the receiver are sent for estimation such as in BLAST system and others[5]. In this paper, we apply a Markov Chain Monte Carlo (MCMC) method, which has been widely used in design of blind Bayesian receivers[6,7], to jointly estimate the CSI and the transmitted signal semi-blindly. A few pilot symbols are needed here for distinguishing the transmitting antennas.

This paper is organized as follows. Section II describes the system model of MIMO system. Section III introduces the semi-blind channel estimation algorithms based on MCMC methods. Section IV uses simulations to demonstrate the effectiveness of our algorithms. Section V gives the conclusions.

II. MIMO System Model

Let us consider a discrete-time MIMO system without Inter-Symbol Interference (ISI). Assuming that there are $M_r$ receiving antennas and $M_t$ transmitting antennas in this system, the $i$-th received signal at time $n$ is given by

$$y_{i,n} = \sum_{j=1}^{M_t} h_{i,j} x_{j,n} + w_{i,n}$$  \hspace{1cm} (1)

where $x_{j,n}$ is the $j$-th transmitted signal and $w_{i,n}$ is the additive noise on the $i$-th received signal at time $n$, $h_{i,j}$ is the complex channel gain from the $j$-th transmitting antenna to the $i$-th receiving antenna.

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Considering a frame of total $NM_t$ transmitted symbols, we can rewrite Eq.(1) in a matrix form as

$$Y = HX + W$$  \hspace{1cm} (2)

where

- $Y = [y_1^T \ y_2^T \ldots \ y_{M_r}^T]^T$, $y_i = [y_{i,1} \ y_{i,2} \ldots \ y_{i,N}]$ is the matrix of the received signal;
- $X = [x_1^T \ x_2^T \ldots \ x_{M_t}^T]^T$, $x_i = [x_{i,1} \ x_{i,2} \ldots \ x_{i,N}]$ is the matrix of the transmitted signal;
- $W = [w_1^T \ w_2^T \ldots \ w_{M_r}^T]^T$, $w_i = [w_{i,1} \ w_{i,2} \ldots \ w_{i,N}]$ is the matrix of additive channel noise, and

$$H = [h_{1,1} \ h_{1,2} \ldots \ h_{1,M_t}]$$

$$[h_{2,1} \ h_{2,2} \ldots \ h_{2,M_t}]$$

$$\vdots \quad \vdots \quad \vdots$$

$$[h_{M_r,1} \ h_{M_r,2} \ldots \ h_{M_r,M_t}]$$

All of variables in Eq.(2) may be complex. In this paper, we take following assumptions:

1. The transmitted symbols are $x_{i,k} \in \{-1, 1\}$, which means using a BPSK modulation scheme.
2. The channel's additive noise $w_{i,n}$ is i.i.d. complex Gaussian variable with a probability density function (pdf) of

$$p(w_{i,k}) = \frac{1}{2\pi\sigma_n^2} \exp\left(-\frac{1}{2\sigma_n^2}|w_{i,k}|^2\right)$$  \hspace{1cm} (3)

The variance of noise is assumed being known to the receiver.

3. $H$ remains constant in a frame and may vary between different frames. Its element $h_{i,j}$ is i.i.d. complex Gaussian variable with a pdf of

$$p(h_{i,j}) = \frac{1}{2\pi\sigma_h^2} \exp\left(-\frac{1}{2\sigma_h^2}|h_{i,j}|^2\right)$$  \hspace{1cm} (4)

The system's average Signal to Noise Ratio (SNR) is defined as

$$\text{SNR}_{\text{average}} = \frac{M_r \sigma_n^2}{\sigma_h^2}$$  \hspace{1cm} (5)

If the estimation of $H$ is $\hat{H} = \{\hat{h}_{i,j}\}_{M_r \times M_t}$, then we define a normalized channel estimation error as

$$\delta H = \frac{E \left\{ \sum_{i=1}^{M_r} \sum_{j=1}^{M_t} |h_{i,j} - \hat{h}_{i,j}|^2 \right\}}{2M_rM_t \sigma_h^2}$$  \hspace{1cm} (6)

### III. Channel Estimation Algorithms

In this section, we first introduce the principle of MCMC methods, then deduce the conditional *a posteriori* distributions that we will need in our algorithms and give reasons for the need of some pilot symbols in our system. Finally, we give the detail of channel estimation algorithms and analyze their computation complexity.

#### 1. Basics of MCMC methods

MCMC methods such as the Gibbs sampler and Metropolis algorithm\[^8\] are powerful simulation-based techniques for sampling from high-dimensional and/or non-standard probability distributions. Recently, they have been applied to the design of blind Bayesian