RETROSPECTIVE RESERVES FOR THE INSURANCES OF THE PERSON IN THE FRAMEWORK OF MULTISTATE MODELS*

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Retrospective reserve is usually meant as the expected accumulated value at a given time of past premiums less benefits. With reference to multistate models, both the state of the policy at the time of valuation and its past story must be considered in calculating such expected value. Since it is unlikely that the retrospective reserve can be evaluated for any possible path of the risk, different definitions can be given according to the hypotheses on its story.

In this paper we firstly examine the main definitions of retrospective reserve given in the framework of multistate models. Then, we formulate a new definition which generalizes that given in conventional life insurance mathematics. Comparisons among the various definitions are proposed in some examples which, inter alia, show the different behaviour and magnitude of such reserves.

1. Introduction

Retrospective reserve is usually meant as the expected accumulated value at a given time of past premiums less benefits. Such expected value is conditional on some information available at the time of calculation.

In single life insurance, the only possible information relate to the existence of the contract at that time; hence, there is just one way of defining the retrospective reserve, i.e. as the expected accumulated value of past premiums less benefits given the insured is alive.

On the contrary, in multistate models information concern both the state of the contract at the time of calculation and its past story. So, we can give as many definitions of retrospective reserve as the ways of summarizing past behaviour of the policy.

In this paper, with reference to a time-continuous Markov model (briefly

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described in Section 2), we examine the definitions of retrospective reserve given in literature (Section 3) and we formulate a new definition which generalizes that given in conventional life insurance mathematics (Section 4). Comparisons among the various definitions are proposed in some examples (Section 5).

2. The Markov model. Payment functions. Prospective reserve

(a) The Markov model. Consider a life insurance policy; when a multistate approach is adopted, its evolution is represented as a sample path of a time-continuous, inhomogeneous Markov chain with a finite state space \( S \),

\[ S = \{1, 2, ..., N\}. \]

Let \( S(t) \) be the (unknown) state of the policy at time \( t \) in the period \([0, u]\) of the coverage and assume that \( S(0) = 1 \). We denote transition probabilities by \( P_{ij}(t, u) \) and transition intensities by \( \mu_{ij}(t) \). We assume that such functions are well-defined for all the operations to be performed.

The seminal contributions to multistate modelling in life insurance are given by Hoem [1] and Hoem [2], where several aspects are examined (such as premiums and prospective reserves, as well as safe-side requirements and retrospective reserves). An overall perspective on multistate modelling is also given by Wolthuis [6].

It must be pointed out that when life insurance is concerned, the most natural setting is time-continuous. Actually, a time-continuous Markov approach is generally adopted in the multistate framework. However, because of the structure of statistical data, time-discrete models might be required. A description of time-discrete multistate models is given by Wolthuis [6].

(b) Benefits and premiums. Suppose that the policy entitles the insured to receive a continuous annuity benefit at a rate \( b_j(t) \) at time \( t \) if \( S(t) = j \) and a lump sum \( c_{jk}(t) \) at time \( t \) if a transition occurs from state \( j \) to state \( k \) at that time. Conversely, we assume that the insured pays continuous premiums at a rate \( p_j(t) \) at time \( t \) if \( S(t) = j \) (for the sake of simplicity we will not consider discrete payments; nevertheless, the results discussed below also hold when a finite number of discrete payments is involved).

Let \( \mathcal{P}_j(t, u) \) and \( \mathcal{B}_j(t, u) \) be, respectively, the actuarial value in \( t \) of premiums and benefits paid in the interval \([t, u]\), given \( S(t) = i \). We assume that such actuarial values are calculated with the deterministic discount function \( v = e^{-\delta} \), where \( \delta \) is the force of interest. So we have

\[
\mathcal{P}_j(t, u) = \int_t^u v^{s-t} \sum_j P_{ij}(t, s) p_j(s) \, ds;
\]

\[
\mathcal{B}_j(t, u) = \int_t^u v^{s-t} \sum_j P_{ij}(t, s) b_j(s) \, ds + \int_t^u v^{s-t} \sum_j P_{ij}(t, s) \sum_{k \neq j} \mu_{jk}(s) c_{jk}(s) \, ds.
\]