RADIIATION FLUX DISTRIBUTION OVER SOLAR IMAGES FORMED ON THE FOCAL PLANE BY A PARABOLOIDAL REFLECTOR WITH TRACKING ERRORS

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Abstract—The radiation flux distribution of off-centered solar images formed by a sun-tracking paraboloidal reflector is theoretically analyzed for several tracking error angles using Jose's sunshape equation and assuming a specularly reflecting surface without taking meteorological conditions into consideration. The results are printed out by computer in the form of shade density maps to bring out a clear contrast to the low and high flux areas of a full image.

INTRODUCTION

Radiation flux analyses of the solar images thrown on the focal plane by paraboloidal reflectors have been studied by Jose [1], Harris and Duff [2] and others. Jose's work is particularly worthy of noting for his sunshape equation which he determined from studying the actually measured intensity data. His analysis, however, seems inconclusive in some area of the image by failing to give full account of all the contributions by the relevant beams contained in the reflected radiation cones. He also limited his analysis within the focal spot bounded by the image produced by the reflected cone from the vertex of the paraboloidal reflector.

Harris and Duff calculated the flux for a real surface. They incorporated the Jose's sunshape function with the idea of randomly varying surface normals whose deviations from the reference normal were characterized by a bivariate probability density function. This idea was originally proposed by Pettit [3] and also by Butler and Pettit [4]. Biggs and Vittitoe [5] proposed the use of a variable sunshape function to allow for the meteorological conditions in their flux calculations.

Recently, Look and Sundvold [6] introduced a procedure of calculating the radiative energy leaving a point source being reflected from one surface, and striking another. This procedure was applied to regular and extended parabolic and also circular cylindrical surfaces. The source radiation they used consisted simply of parallel beams with distributed wave lengths. In stead of probabilistic scattering of radiation by the surface, they took the reflectivity as a function of wave length.

None of the workers named above worked on obliquely incident radiations. In this work, we have been working on a two-dimensional sun-tracking system to orient a paraboloidal reflector towards the sun. The penumbra sun sensor mounted on the rim of the reflector has a resolution of 0.3 degrees, giving at least that much tracking error. Errors can be introduced in the assembling stage of the system due to machining errors and allowances. These errors will make the incident radiation oblique to the axis of the reflector. This work is proposed for analyzing the radiation flux on the focal plane when such radiations are reflected from a paraboloidal reflector. The tracking errors may be variable both in direction and in magnitude. They are likely to follow a certain pattern depending on the tracking system. Since this pattern is not known at this stage, we will choose several probable tracking error angles and concentrate ourselves in calculating the image flux density.

VECTOR REPRESENTATION

Visualize a paraboloidal reflector such as shown in Fig. 1. The dimensionless equation of a paraboloid can be written as

\[ 4\phi Z = X^2 + Y^2 \]  

(1)

where \( \phi \) is the dimensionless focal length \( f/R \), and \( X, Y \), and \( Z \) are the dimensionless coordinates \( x/R \), \( y/R \) and \( z/R \) in the directions of \( x \), \( y \), and \( z \), respectively. Suppose a point \( O(X_0, Y_0, Z_0) \) on the reflecting surface, then, the radiation beam incident to and reflected from the point, as well as the normal to the paraboloid at the same point, can be expressed by vector notations \( \vec{A}, \vec{B}, \) and \( \vec{P} \)

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among which the relation below holds:

$$\mathbf{\hat{B}} = -2 (\mathbf{\hat{A}} \cdot \mathbf{\hat{P}}) \mathbf{\hat{P}} + \mathbf{\hat{A}}.$$  \hspace{1cm}(2)

Therefore, the incident vector $\mathbf{\hat{A}}$, through reflection is transformed into vector $\mathbf{\hat{B}}$, and this vector will intersect the focal plane, $Z = \phi$, at the point whose coordinates are:

$$X = X_i + \left( \frac{b_1}{b_3} \right) (\phi - Z_i),$$

$$Y = Y_i + \left( \frac{b_2}{b_3} \right) (\phi - Z_i),$$

and $Z = \phi$  \hspace{1cm}(3)

where $b_1$, $b_2$, and $b_3$ are the X, Y and Z-directional components of the reflected unit vector $\mathbf{\hat{B}}$.

Suppose that the angle formed between the incident radiation and the reflector axis is $\sigma$ as shown in the inset of Fig. 1, and the angle formed between the X-directional tangent at point $O_1$ and the X-Y plane is $\delta$, and that formed between the incident beam and the normal to the reflector surface at point $O_1$ is $\epsilon$, then, the energy $\delta E_\phi$ that falls on a small area $\delta S_\phi$ of the reflector at $O_1$ can be expressed, if $I_o$ is the incident intensity, by

$$\delta E_\phi = I_o \cos \epsilon \cos \delta \delta S_\phi$$  \hspace{1cm}(4)

where $\delta S_\phi$ is the projected area of $\delta S_\phi$ on the X-Y plane.

Denoting the unit vector in axial direction by $\mathbf{\hat{S}}$,  

$$\cos \epsilon = -\mathbf{\hat{A}} \cdot \mathbf{\hat{P}},$$  \hspace{1cm}(5a)  

and  

$$\cos \delta = \mathbf{\hat{S}} \cdot \mathbf{\hat{P}}.$$  \hspace{1cm}(5b)

So far, the incident radiation on point $O_1$ was treated as a single beam, while actually solar radiation falls on a point forming an inverted cone of light with an angle width of 0.53 degrees as shown by $a, b$ in Fig. 2. Thus solar images are formed by reflected cones by being slantly intercepted by the focal plane. The images appear usually as somewhat distorted ellipses in shape instead of a point as is true in the case of parallel beams. Therefore, the brightness of the image, or the flux, at a point on the focal plane must be obtained by summing up the effects of all such reflected cones from all influential points on the reflector.

**RADIATION FLUX ON THE FOCAL PLANE**

In order to analyze the radiation flux on the focal plane, the following assumptions were postulated:

1. The radiation intensity of the apparent disk of sun is not uniform: it is distributed in accordance with the Jose’s sunshape function.
2. The reflector surface is optically perfect and geometrically an ideal paraboloid.
3. Sun tracking is not perfect: the central beam of the incident radiation cone is slightly oblique to the axis of the reflector.
4. The conditions of the earth’s atmosphere are the same as those of any fine days.
5. The reflector area shaded by the receiver is negligible.

The empirical equation of sunshape that Jose [1] presented is as follows:

$$I_c = R_c + 1.5641 \sqrt{R_c^2 - r_c^2} I_{c_0}$$  \hspace{1cm}(6)

where $I_{c_0}$, the intensity at the center of the cone, is determined as a function of $R_c$, the radius of the apparent...