NONLINEAR REGIMES OF CONVECTION OF AN ELASTICOVISCOUS FLUID IN A CLOSED CAVITY HEATED FROM BELOW

E. N. Krapivina and T. P. Lyubimova

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The two-dimensional thermal convection of a elasticoviscous fluid in a horizontal cylinder with a square cross-section heated from below is investigated. For describing the rheological properties of the fluid the Oldroyd model with the upper convective derivative is used. The investigation is carried out numerically using the finite-difference method. The instability limits of mechanical equilibrium with respect to monotonic and oscillatory perturbations are determined. Supercritical convection regimes are investigated numerically.

The mechanical equilibrium of a Newtonian fluid heated from below is stable only for a relatively small temperature difference. An increase in the temperature gradient leads to instability, loss of stability occurring against a background of monotonic perturbations. There is a qualitative difference between the behavior of fluids with a complex rheology, heated from below, and the behavior of Newtonian fluids.

In the case of a dilatant power-law fluid, for an arbitrarily small temperature gradient the development of perturbations leads to a steady-state finite-amplitude motion. In the case of a pseudoplastic, for Rayleigh numbers larger than a certain threshold value convection is rigidly excited (hard excitation) [1]. There are many studies which consider the linear stability of equilibrium of a horizontal viscoelastic or elastoviscous fluid layer heated from below (see, for example, [2, 3]). In [2] for describing the rheological properties of the fluid the Maxwell model is used and in [3] the Oldroyd model. It is shown that the presence of elastic properties of the fluid leads to the existence of two types of instability, namely, with respect to monotonic and oscillatory perturbations. Loss of stability of equilibrium with respect to monotonic perturbations occurs for the same values of the Grashof number as in a Newtonian fluid. However, there appears a region of the parameters in which oscillatory perturbations are the most dangerous.

In the present paper we investigate numerically the convective stability of equilibrium and supercritical convective motions of an elasticoviscous fluid heated from below in a closed cavity. We determine the stability limits by solving the linearized equations of the small perturbations numerically by the finite-difference method and study the supercritical convective motions by solving the problem numerically in the complete nonlinear formulation.

1. FORMULATION OF THE PROBLEM. METHOD OF NUMERICAL SOLUTION

We will consider the plane convective motion of an elasticoviscous fluid in a square cavity with side $h$. The $x$ and $y$ axes are directed along the lower boundary and the left wall of the cavity, respectively. The boundaries of the region are rigid, and on the horizontal boundaries constant different temperatures are maintained (the temperature of the upper boundary is chosen as the point of reference, the difference between the upper and lower boundary temperatures being equal to $\theta$). The vertical boundaries are thermally insulated.

In order to describe the rheological properties of the fluid we will use the Oldroyd "B" model [4]

$$\begin{align*}
\sigma_{ij} + \lambda \sigma'_{ij} &= v D_{ij} + \lambda D'_{ij} \\
D_{ij} &= \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}
\end{align*}$$

Here, $\sigma_{ij}$ is the stress tensor, $v$ is the kinematic viscosity coefficient, $D_{ij}$ is the strain-rate tensor, $\lambda$ is the lag, $\lambda$ is the relaxation time. $\sigma'_{ij}$ and $D'_{ij}$ are convective derivatives, for example,

$$
D'_{ij} = \frac{\partial D_{ij}}{\partial \tau} + \nu \frac{\partial D_{ij}}{\partial \tau} - \frac{\partial v_i}{\partial x_m} D_{mj} - \frac{\partial v_j}{\partial x_m} D_{im}.
$$

We will write the thermal convection equation in the Boussinesq approximation in terms of the stream function $\psi$ and the $z$-component of the velocity vortex $\varphi$. 


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\[ v_x = \partial \psi / \partial y, \quad v_y = -\partial \psi / \partial x, \quad \varphi = \text{rot}_x \psi \]

\[ \frac{\partial \sigma_{xx}}{\partial t} + \tau \left( \frac{\partial \sigma_{xx}}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \sigma_{xx}}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \sigma_{xx}}{\partial y} - 2 \frac{\partial^2 \psi}{\partial x \partial y} \sigma_{xx} - 2 \frac{\partial^2 \psi}{\partial x^2} \sigma_{xy} \right) = 0 \]  

\[ \frac{\partial \sigma_{xy}}{\partial t} + \tau \left( \frac{\partial \sigma_{xy}}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \sigma_{xy}}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \sigma_{xy}}{\partial y} + 2 \frac{\partial^2 \psi}{\partial x \partial y} \sigma_{xy} + 2 \frac{\partial^2 \psi}{\partial y^2} \sigma_{xy} \right) = 0 \]  

\[ \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} + \tau \left( \frac{\partial D_{xx}}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial D_{xx}}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial D_{xx}}{\partial y} + 2 \frac{\partial^2 \psi}{\partial x \partial y} D_{xx} + 2 \frac{\partial^2 \psi}{\partial x^2} D_{xy} \right) = 0 \]  

\[ D_{xx} = 2 \frac{\partial^2 \psi}{\partial x \partial y}, \quad D_{xy} = -\frac{\partial^2 \psi}{\partial x \partial y}, \quad D_{yy} = \frac{\partial^2 \psi}{\partial y^2} \]

Equations (1.1)–(1.4) are written in dimensionless form. As the units of measurement of the velocity, time, and temperature we chose \( v/h, h^2/v, \) and \( \theta \), respectively. The governing parameters of the problem are the dimensionless lag and relaxation times \( \tau_x \) and \( \tau_y \), and the Grashof and Prandtl numbers \( Gr \) and \( Pr \):

\[ \tau_x = \lambda_x v / h^2, \quad \tau_y = \lambda_y v / h^2, \quad Gr = g \beta \theta h^3 / v^2, \quad Pr = v / \chi \]

The hydrodynamic and thermal boundary conditions have the following form:

\[ \psi = \partial \psi / \partial x = 0, \quad \partial T / \partial x = 0 \quad (x = 0, 1) \]

\[ \psi = \partial \psi / \partial y = 0, \quad T = 0 \quad (y = 1) \]

\[ \psi = \partial \psi / \partial y = 0, \quad T = 1 \quad (y = 0) \]

In the state of mechanical equilibrium there is no motion and the temperature gradient is constant

\[ \psi_x = 0, \quad T_x = 1 - y \]

We will consider the stability of equilibrium with respect to small perturbations

\[ \psi = \psi_0 + \psi', \quad T = T_0 + T' \quad \varphi = \varphi_0 + \varphi', \quad \sigma_{xx} = \sigma_{xx_0} + \sigma_{xx}' \]