INFLUENCE OF A SURFACTANT ON THE INSTABILITY OF A FALLING LIQUID FILM

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The hydrodynamic instability of a film flow of a weak solution containing a soluble volatile surfactant is investigated. Diffusion of the surfactant in the liquid, its evaporation into the boundary gas medium, and the adsorption and desorption processes in the near-surface layer are taken into account. A system of evolutionary equations is derived and a steady-state solution film flow along a vertical surface and the stability of this flow are investigated for the simultaneous action of body and capillary forces and the Marangoni effect. Hydrodynamic and diffusion instability modes are detected and their properties are investigated for constant and variable surfactant concentration in the adsorbed sublayer.

1. SYSTEM OF EVOLUTIONARY EQUATIONS FOR A FALLING FILM WITH A SURFACTANT

Let a viscous liquid film flow downward along a vertical wall under the action of the gravity force. The liquid is a weak solution containing a soluble volatile surfactant. The surfactant distribution over the film is governed by the diffusion of the surfactant in the liquid, the adsorption-desorption processes between the liquid and the adsorbed surface layer, and evaporation of the surfactant from the surface into the boundary gas medium.

We introduce the coordinate system $x, y$ with the origin at the wall and the $x$ axis aligned with the gravity force vector. At a moment $t$ at an arbitrary point occupied by the liquid the state of the falling solution can be described by means of the following variable quantities: the pressure $p$, the velocities $U$ and $V$, the volume surfactant concentration $C$, and the adsorbed-layer surfactant concentration $\Gamma$. The state of the entire film can be described by the local thickness $h$. We will consider only plane motion of the liquid. Then $h$ and $\Gamma$ depend only on $x$ and $t$ and the other variables depend on $x, y,$ and $t$. We introduce the characteristic constant scales $l, U, C, \Gamma$ and define the dimensionless variables $\chi, \gamma, h, t, \rho, U, V, C, and \Gamma$ by means of the relations

$$
\begin{align*}
x, y, h, t, \rho & \rightarrow \chi\sqrt{k}, \gamma\sqrt{l}h, t/kU, \rho U^2/l \quad (1.1) \\
U, V, C, \Gamma & \rightarrow U/l, kU^2V, (1 + C), (1 + \Gamma)
\end{align*}
$$

In dimensionless variables the Navier-Stokes, continuity, and diffusion equations and the boundary conditions on the wall and the free surface have the form:

$$
\begin{align*}
U_x + UU_x + VU_y = -p_x + \frac{1}{kRe}(U_{yy} + k^2U_{xx}) + \frac{1}{kFr}
\quad (1.2) \\
k^2(V_x + UV_x + VW_y) = -p_y + k^2\frac{1}{kRe}(V_{yy} + k^2V_{xx})
\quad (1.3) \\
U_x + V_y = 0 \\
C_x + UC_x + VC_y = \frac{1}{kPe}(C_{xx} + k^2C_{yy})
\quad (1.4)
\end{align*}
$$

$$
\begin{align*}
y=0: & \quad U=0, \quad V=0, \quad C_y=0 \\
y=h(x, t): & \quad h_x + Uh_t = V \quad (1.5)
\end{align*}
$$

\[
p = \frac{k^2}{W_e} \left( 1 - \frac{\text{Ma} \cdot \text{We} \cdot C}{\text{Re}} \right) \hat{h}_0 - \frac{2k^2}{k \cdot \text{Re}} (V_i - U_j h_i - k^2 V_i h_i)
\]

\[
U_j + k^2 (V_j + 4h_j V_j) = -k \text{Ma}(C_j + h_j C_j)
\]

\[
\text{Bi}(1 + C) + kG(\Gamma_i + U\Gamma_i + U \Gamma_j + U_j) - k^2 \text{Di} \Gamma_{ii} + C_i = 0
\]

\[
\pi_i (1 + C_i) - \pi_i (1 + \Gamma) = -C_i
\]

\[
\text{Re} = \frac{U_i l}{v}, \quad \text{Fr}^2 = \frac{g l}{U_i^2}, \quad \text{Pe} = \frac{U_i l}{D}, \quad \text{Ca} = \frac{\mu U_i}{\sigma}
\]

\[
\text{Bi} = \frac{k_c l}{D}, \quad G = \frac{\Gamma U_i}{C D}, \quad \text{Di} = \frac{D_j \Gamma_{ij}}{l D C_i}, \quad \text{Ma} = -\frac{C_i}{\mu U_i} \left( \frac{d \sigma}{d C_i} \right)
\]

Here, \( \rho \) is the density, \( \mu = \nu p \) and \( D \) are the viscosity and diffusion coefficients of the solution, \( g \) is the acceleration of gravity, \( \sigma \) and \( D_i \) are the surface tension and diffusion coefficients, and \( K_{\text{e}}, K_{\text{a}}, \) and \( K_{\text{f}} \) are the evaporation, adsorption, and desorption coefficients, respectively. The dilatation coefficient \( k \) was introduced in (1.1) to smooth out the order of derivatives of the hydrodynamic functions with respect to the variables \( x, y \) in (1.2)-(1.6). In the case of wave film flows, small values of \( k \) correspond to long waves.

The formulation of the problem (1.2)-(1.6) differs from that of study [1] due to the variable concentration in the absorbed sublayer \( \Gamma = \Gamma(x, t) \). Its value is determined on the basis of the adsorption barrier concept [2, 3]. The surface adsorption-desorption kinetics are specified by the last of equations (1.6). A similar model was used in [4] to investigate the stability of a fixed layer on a horizontal surface.

The surface tension coefficient is assumed to be a linear function of the concentration \( c \) on the surface of the layer

\[
\sigma = \sigma_0 + (d \sigma / d c)(c - c_p)
\]

and hence the Marangoni number \( \text{Ma} = \text{const} > 0 \) for the motions of the fluid considered. There also exist surfactant solutions for which \( \text{Ma} < 0 \).

In this problem, the Marangoni number is the critical parameter determining the interaction between the hydrodynamic and diffusion processes in the layer.

Another approach to the problem considered is to assume the surface tension to be a linear function of the concentration \( \Gamma \) in the adsorbed layer and set

\[
\sigma = \sigma_0 + (d \sigma / d \Gamma)(\Gamma - \Gamma_0)
\]

\[
\text{Ma} = -\frac{\Gamma_{ij}}{\mu U_i} \left( \frac{d \sigma}{d \Gamma} \right)
\]

We will investigate the boundary value problem (1.2)-(1.6) using the methods of studies [5, 6]. Earlier, in [7, 8] on the basis of this method a theory of linear and nonlinear waves in a homogeneous liquid film with a given constant or variable surface friction was constructed. The use of this method to analyze the Marangoni effect initiated by heat transfer in the film was discussed in study [9]. We choose \( U_i \) and \( k \) so that [5, 6] the coefficients of the viscous, gravity, and capillary forces in the first of equations (1.2) are of the same order and normalized by the parameter \( \delta \)

\[
\frac{k^2}{W_e} = \frac{3}{k \cdot \text{Re}} = \frac{1}{k \cdot \text{Fr}} = \frac{1}{5 \delta}
\]

From (1.7) and (1.10) we derive the following relations:

\[
l = (3v^2 / g)^{1/3} \text{Re}^{1/3}, \quad U_i = (g v / 3)^{1/3} \text{Re}^{2/3}, \quad \gamma = \frac{\sigma}{\rho} (v^2 / g)^{1/3}
\]

\[
k = \gamma^{-1/3} (3 \text{Re})^{2/3} = (3 \text{Ca})^{1/3}, \quad \delta = \frac{1}{45} \gamma^{1/3} (3 \text{Re})^{1/3}
\]