STABILITY OF A VISCOUS SUBSONIC SWIRL FLOW

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The results of calculating the stability of a three-dimensional swirl flow of a viscous heat-conducting gas are presented. The stability characteristics are determined using the linear time-dependent theory of plane-parallel flow stability. The main undisturbed axisymmetric vortex flow was determined numerically using a quasi-cylindrical approximation for the complete set of Navier-Stokes equations. The circulation of the peripheral velocity in the cocurrent flow surrounding the viscous vortex core was assumed to be constant. In analyzing the stability, nonaxisymmetric perturbations in the shape of waves traveling along the vortex axis with both positive and negative wavenumbers were considered; in these two cases the perturbation rotation is either the same or opposite in sense to the rotation in the vortex core. Neutral stability curves are determined for various values of the swirling parameter and the cocurrent flow Mach number.

The mathematical simulation of the formation, dynamics, and stability of swirl flows, as well as the breakdown of vortex flows, first discovered experimentally in [1, 2], is one of the most important problems of fluid dynamics. From a practical standpoint, the analysis of the stability of vortex flows of various types, such as tornados in the atmosphere and trailing vortices shed from aircraft wings, their interaction with the ambient medium and subsequent breakdown is of the most interest.

Vortex flow stability depends not only on the peripheral velocity distribution but also on the axial velocity whose distribution in the vicinity of the vortex axis can be of either the jet or wake type. In most studies of swirl flow stability [3--10] carried out within the framework of the incompressible fluid model, undisturbed velocity profiles given by the well-known Q-model, a simplified version of the Batchelor vortex model for an incompressible fluid [5, 6], were used.

In actual fact, the vortex flow dynamics and stability characteristics can change considerably when the medium compressibility and heat transfer are taken into account [11, 13]. The present study deals with the influence of the thermal conductivity of the gas and the cocurrent flow Mach number on the linear stability characteristics for the swirl flow in a viscous vortex core surrounded by a cocurrent subsonic flow in the case in which the peripheral velocity circulation is constant outside the vortex core.

1. FORMULATION OF THE PROBLEM

Consider the stability of a steady-state axisymmetric swirl flow of viscous heat-conducting gas surrounded by a cocurrent subsonic flow. The flow outside the viscous vortex core is assumed to be uniform and have a constant peripheral velocity circulation. We will use cylindrical coordinates \(x', y', \phi'\) and the corresponding velocity components \(v_x', v_y', \text{ and } v_{\phi}'\) and assume that the longitudinal \(x\) axis is aligned with the vortex axis.

In order to determine the parameters of the main undisturbed flow, we will use the dimensionless coordinates \(x'=\ell x, r'=\ell r, \text{ and } \phi\) and the velocity components \(v_x'=u_0 v_x, v_y'=u_0 v_y, \text{ and } v_{\phi}'=u_0 v_{\phi}\), respectively. It is convenient to use the following dimensionless variables for the density, dynamic viscosity, pressure, and enthalpy: \(\rho'=\rho_0\rho, \mu'=\mu_0\mu, P'=\rho_0 u_0^2 P, \text{ and } h'=u_0^2 h\). Here, \(u_0, \rho_0, \mu_0, \text{ and } h_0\) are the longitudinal velocity, density, dynamic viscosity, and enthalpy, respectively. The longitudinal length \(\ell\) specifies the scale at which the flow in the viscous vortex core develops. For a trailing vortex shed from a finite-aspect-ratio wing, this scale is determined by the distance at which the vortex sheet forming the outer inviscid stream with a constant peripheral velocity circulation is rolled up, while a viscous vortex core with corresponding peripheral and longitudinal velocity profiles is formed in the vicinity of the vortex filament axis. The corresponding Reynolds number may be represented as \(Re=\rho_0 u_0 \ell /\mu_0\).

At high Reynolds numbers, \(Re \gg 1\), the viscous vortex core thickness is small and, as a rule, is controlled by the thickness of the boundary layer on the vortex-generating body. Outside the viscous core, in the region where \(x \sim O(1)\) and \(r \sim O(1)\), the flow may be assumed to be slightly perturbed. To describe the flow in the viscous core, we introduce a new variable \(V_0=v_{\phi}'\) which represents the peripheral velocity circulation divided by \(2\pi\). At high Re, in the viscous flow region, it is convenient to introduce the new variables...
The system of quasi-cylindrical approximation equations governing the viscous heat-conducting gas flow in a vortex core is similar to the system used to describe incompressible swirl flows [6 - 10]. Together with the associated boundary conditions, this system has the form:

\[
\rho \frac{\partial V_x}{\partial x} + \rho \frac{\partial V_x}{\partial r} = -\frac{\partial p}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \mu \frac{\partial V_x}{\partial x} \right) + \frac{\mu}{r} \frac{\partial V_x}{\partial r} \right] - \alpha_0 \frac{\partial V_x}{\partial x} - \frac{\beta}{r} \frac{\partial V_x}{\partial r}
\]

\[
\rho \frac{\partial V_\phi}{\partial x} + \rho \frac{\partial V_\phi}{\partial r} = \left[ \frac{\partial}{\partial x} \left( \mu \frac{\partial V_\phi}{\partial x} \right) - \frac{\mu}{r_1} \frac{\partial V_\phi}{\partial r_1} - \frac{2V_\phi \partial \mu}{r_1} \right] + \frac{\partial p V_\phi}{r_1} - \frac{\rho V_\phi}{r_1} + \frac{\partial p V_r}{\partial r_1} = 0
\]

\[
x = 0: \quad V_x = V_{ho}(r_1), \quad V_\phi = V_{ho}(r_1), \quad h = h_0(r_1)
\]

\[
r_1 = 0: \quad V_r = 0, \quad \frac{\partial V_x}{\partial r_1} = 0, \quad V_\phi = 0, \quad \frac{\partial h}{\partial r_1} = 0
\]

\[
r_1 \to \infty: \quad V_x = V_{ho}(x), \quad V_\phi = V_{ho}(x) = \Gamma = \text{const}, \quad h = h_0(x)
\]

Outside the viscous region, the flow is assumed to be isentropic and uniform or to have a nonzero longitudinal pressure gradient; in the latter case, the longitudinal velocity, enthalpy, pressure, and Mach number are functions of the longitudinal coordinate \(x\): \(V_x = V_{ho}(x), h = h_0(x), p = p(x), \) and \(M = M_0(x)\). A power-law viscosity-temperature relationship is assumed: \(\mu/\mu_0 = (h/h_0)^\gamma\). Here, \(M\) is the Mach number, \(\sigma\) is the Prandtl number, and \(\gamma\) is the specific heat ratio. In the calculations \(\sigma = 0.75, \omega = 0.7,\) and \(\gamma = 1.4\).

To integrate the system of equations (1.2), we used the finite-difference method described in detail in [12, 13]. In the initial section \(x = 0\) velocity profiles analogous to those of [4 - 6, 9, 10] were preassigned:

\[
V_{ho} = 1 + \Delta \exp(-\beta r_1^2), \quad V_{ho} = \Gamma(1 - \exp(-\beta r_1^2)
\]

The initial velocity distribution corresponds to that in a \(Q\)-vortex in a cocurrent stream [4, 9]; moreover, the peripheral velocity \(V_\phi\) profile is identical to that in the Lamb-Oseen vortex model [16]. The parameter \(\Delta\) makes it possible to obtain various longitudinal velocity profiles near the vortex axis; thus, \(\Delta = 0\) corresponds to a uniform flow in the initial section, \(\Delta > 0\) to a jet-like swirl flow, and \(\Delta < 0\) to a wake-like vortex flow. The parameter \(\Gamma\) specifies the peripheral velocity circulation. The enthalpy distribution in the initial section was assumed to be uniform: \(h_0(x = 0, r_1) = h_0(x = 0)\). In the calculations the Reynolds number was taken to be \(Re = 300\). In studying the swirl flow stability we used the undisturbed flow parameters (velocity and enthalpy profiles) obtained by integrating Eqs. (1.2) numerically; it was assumed that in each section \(x = \text{const}\) these parameters do not depend on the longitudinal coordinate.

To formulate the linear stability problem for a swirl flow, we will use the Navier-Stokes equations for a viscous heat-conducting gas written in cylindrical coordinates \((x', r', \phi)\). We will write down the equations for infinitesimal