EFFECT OF SURFACE AND INTERNAL WAVES ON THE HYDRODYNAMIC CHARACTERISTICS OF A CONTOUR IN THE LINEAR APPROXIMATION

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A method of solving the problem of the motion of an elliptic contour in a three-layer fluid is developed within the framework of the linear theory. The results of calculating the hydrodynamic contour loads and the shape of the interfaces are presented for the following problems: the motion of a contour beneath an interface between two media and in a two-layer fluid both beneath a rigid lid and a free surface. On the basis of the numerical experiment it is concluded that surface and internal waves have a significant effect on the hydrodynamic characteristics of the contour.

Many authors have long been investigating surface and internal waves generated by a body submerged in a fluid. The most extensive review of this research is contained in [1]. In addition, we note the following studies. In [2] an original method of solving the problem of two-layer fluid flow past a circular cylinder was proposed. The solution was represented in the form of a rapidly converging series, formulas for the hydrodynamic loads were obtained, and values of the wave drag and lift force were given. The review [3] contains the results of a numerical solution of the linear problem of the motion of a cylinder in the neighborhood of a liquid-liquid interface and in a two-layer fluid beneath a rigid lid and beneath a free surface. The results are compared with the investigations of other authors. In [4, 5] in order to solve the problem of the motion of a contour in a three-layer fluid the boundary singularity method was developed. The results of extensive numerical experiments to estimate the effect of interfaces on the wave drag and lift force of a circular cylinder were given.

Despite these results, many problems remain insufficiently investigated. In particular, the behavior of the hydrodynamic characteristics and the shape of the interface in the neighborhood of the critical Froude number is unclear. In addition, it is of interest to work out a method of solving the problem in question, to carry out a numerical experiment and on that basis to draw a conclusion concerning the effect of surface and internal waves on the hydrodynamic characteristics of a contour.

1. Let us consider the linear problem of the uniform motion of an elliptic contour L in a three-layer fluid. In each layer $D_k$ ($k=1, 2, 3$) the fluid is assumed to be ideal, incompressible, heavy, and homogeneous. At infinity ahead of the contour the fluid velocities are identical in all the layers. We introduce an inertial coordinate system moving with the contour, the x axis being located along the undisturbed lower boundary of the second and third layers. The ellipse is located in the middle layer. We introduce the following notation: $g$ is the acceleration of gravity, $\rho_k$ is the density of the fluid in the $k$th layer ($k=1, 2, 3$), $V_\infty$ is the velocity of the fluid at infinity ahead of the contour, $a$ and $b$ are the major and minor semiaxes of the ellipse, $h$ is the distance from the center of the ellipse to the interface between the second and third layers, and $H$ is the thickness of the middle layer.

The problem will be considered in the plane of the complex variable $z=x+iy$. We introduce the complex velocities $\overline{V}_k(z)$ ($k=1, 2, 3$) of disturbed motion of the fluid. These velocities are analytic in the $k$th layer (outside $L$ for $k=2$) and must satisfy the following boundary conditions [6]: continuity of the pressure and the normal velocity component across the interface between the media $D_k$ and $D_{k+1}$, no-flow at points on the contour, and the conditions of damping of the disturbed velocities at infinity ahead of the contour:

$$\text{Im} \overline{V}_k(z) = \text{Im} \overline{V}_{k+1}(z) \quad \text{at} \quad z=x+iH(k-2) \quad (k=1, 2)$$

$$\text{Re} \left\{ m_k \frac{d\overline{V}_k(z)}{dz} - m_{k+1} \frac{d\overline{V}_{k+1}(z)}{dz} + i\nu \overline{V}_k(z) \right\} = 0, \quad z=x+iH(k-2) \quad (k=1, 2)$$

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\[ \frac{m_{k+1}}{m_k} = \frac{\rho_k}{\rho_k + \rho_{k+1}}, \quad \frac{m_{k+1}}{m_k} = \frac{v_{k+1}}{v_k} \]

\[ v_k = v m_{k+1} \]

\[ \text{Im} \left( (V_+ + \overline{V}_2(z)) e^{i\theta(z)} \right) = 0 \quad (z \in L) \quad \text{(1.3)} \]

\[ \lim_{z \to -\infty} \overline{V}_k(z) = 0 \quad (k = 1, 2, 3) \quad \text{(1.4)} \]

where \( \theta(z) \) is the angle between the tangent to \( L \) at a point \( z \) and the \( x \) axis.

The functions

\[ \overline{V}_k(z) = \int_L K_k(z, \zeta) \gamma(\zeta) e^{-i\theta(\zeta)} d\zeta \quad \text{(1.5)} \]

must satisfy the boundary conditions (1.1), (1.2), and (1.4). Here, \( \gamma(\zeta) \) is the intensity of the vortex layer simulating \( L \) \((\zeta = \xi + i\eta)\). The expressions \( K_k(z, \zeta) \) \((k = 1, 2, 3)\) are the solution of the corresponding boundary value problem (1.1), (1.2), and (1.4) for a vortex of unit intensity which was obtained in [6] and has the form:

\[ K_1(z, \zeta) = \frac{1}{\pi} \int_0^\infty G_1(\lambda) e^{-i\lambda(z-\zeta)} d\lambda - i \sum_{j=1}^P \text{Res} \left[ G_1(\lambda) e^{-i\lambda(z-\zeta)} \right] \]

\[ K_2(z, \zeta) = \frac{1}{2\pi i} \int_0^\infty \frac{1}{\lambda - \zeta} \int_0^\infty G_2(\lambda) e^{-i\lambda(z-\zeta)} d\lambda - i \sum_{j=1}^P \text{Res} \left[ G_2(\lambda) e^{-i\lambda(z-\zeta)} \right] \]

\[ K_3(z, \zeta) = \frac{1}{\pi} \int_0^\infty G_3(\lambda) e^{-i\lambda(z-\zeta)} d\lambda + i \sum_{j=1}^P \text{Res} \left[ G_3(\lambda) e^{i\lambda(z-\zeta)} \right] \]

\[ G_1(\lambda) = m_{12}^2 \lambda (\lambda m_{23} + v_2 + (\lambda - v_2) e^{-21H}) e^{-21H}/T(\lambda) \]

\[ G_2(\lambda) = \frac{1}{2} \lambda (\lambda m_{23} + v_2 - 21H) (\lambda m_{12} - v_1) e^{-21H}/T(\lambda) \]

\[ G_3(\lambda) = \frac{1}{2} \lambda (\lambda m_{23} - v_2) (\lambda - v_2) e^{-21H} + (\lambda m_{23} + v_2)/T(\lambda) \]

\[ T(\lambda) = (\lambda m_{12} - v_1)(\lambda m_{23} + v_2) + (\lambda - v_2)(\lambda - v_3) e^{21H} \]

where \( \lambda_j \) \((j = 1, P)\) are the positive roots of the equation \( T(\lambda) = 0 \).

\[ P = 2 \quad (A < 0) \quad P = 1 \quad (A > 0) \quad A = m_{23}^2 + (v_2/v_1)m_{12}^2 - H v_2 \]

For determining the function \( \gamma(\zeta) \) we have a singular integral equation obtained by substituting (1.5) in (1.3). After solving this equation by means of the discrete vortex method, we can determine the complex velocities \( \overline{V}_k(z) \) in (1.5) and then the distribution of the pressure \( p \) over the contour \( L \)

\[ p - p_\infty = -\frac{1}{2\pi} \int_1 \left| V_+ + \overline{V}_2(z) \right|^2 - V_0^2 \]

where \( p_\infty \) is the pressure at an infinitely distant point. Then, we can also determine the wave drag \( R_w \), the lift force \( R_y \), and...